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INTERNATIONAL EDUCATION SERIES

HERBART'S A B C OF SENSE-PERCEPTION

AND MINOR PEDAGOGICAL WORKS

TRANSLATED, WITH INTRODUCTION, NOTES, AND COMMENTARY, BY

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TO

THE REVEREND R. BOWEN LOCKWOOD,

AS A MARK OF AFFECTIONATE ESTEEM.

W. J. E.

EDITOR'S PREFACE.

HERBART'S educational theories are in important respects supplementary to those of Pestalozzi. The most important idea is that of apperception. He teaches that the chief object of instruction is to secure the reaction of the mind upon what is offered to sense-perception. We must understand what we see. We must explain it by what we know already. Herbart would secure the assimilation of all our new perceptions by the total amount of experience already stored in our minds. Pestalozzi, on the other hand, made no account of previous experience and of this process of digesting our intellectual food. Pestalozzi wished to have us learn by seeing and hearing and the use of our other senses. In his mental physiology the process of eating is everything, and the process of digestion is ignored. This is the chief defect in his method. Sense-perception is worthy of much attention on the part of the educator, and Pestalozzi's methods have been fruitful of much that is good in our schools, but of far more importance are the processes of apperception—the conversion of sense-impressions into knowledge, and the modification of our previous knowledge by the new experience gained.

The present book contains the larger part of Herbart's writings which deal with Pestalozzi, and will

therefore interest the students of education who desire to study the sequence of these two distinguished educators.

Pestalozzi in his later years came to see that spasmodic efforts in the direction of cultivating sense-perception should give place to a systematic and exhaustive course of training. This led him to inquire what are the elements of sense-perception, and how can an elemental training be given in this activity of the mind. The alphabet leads to a knowledge of letters and the ability to gain knowledge from the printed page—an ability to apperceive printed words. What should be the alphabet of sense-perception? His inquiry into the A B C of Sense-Perception resulted in his doctrine of form, number, and sound or language. We learn through our senses the forms, numbers, and sounds of the objects of the real world. By far the most important item of knowledge gained through sound comes by means of the spoken word. Hence he puts the study of language in school for this item. Arithmetic or number work stands for the second item. Thirdly, the forms of objects could be mastered best by the study of the simple geometric forms and by the art of drawing. These are Pestalozzi's devices.

Herbart thinks over the results of Pestalozzi, and attempts to find a more satisfactory and far-reaching alphabet of sense-perception. Spatial forms and measurements may be studied to best advantage through trigonometry. He accordingly plans a system of instruction which analyzes all forms into triangles and discovers the ratios of the sides of the triangle, one to another, as depending upon the size of the angles. The pupil having formed for himself by measuring a table of natural tangents and secants stated in terms of the radius, is exercised variously in solving problems and discovering the required terms

from the terms given. The two sides of a triangle and its angle being given, what shall be the third side? Two angles and the included side being given, what shall the third angle and the other two sides be?

One can not help admiring the charming manner in which Herbart has reduced to a system this inquiry into the elemental forms in space. He has in this particular made a satisfactory alphabet of sense-perception.

But, as Herbart himself teaches, this alphabet derived from trigonometry does not enable us to spell all the words in sense-perception. There may be an alphabet of color, for instance; another alphabet of musical tones addressed to the ear; an alphabet of tastes; another of odors, and likewise an alphabet for the muscular sense. These, however, are not so important for the interpretation of sense objects as a series of forms to be apperceived through the knowledge of the triangle. But there is to be mentioned still another alphabet which is perhaps of equal or greater importance than this one of trigonometry: it is the alphabet of æsthetic form. Every one who is to live in civilized life should have what is known as good taste in regard to shapes and forms that he makes. It is the fine finish of a piece of work which brings to the labourer an extra price for his production. The knowledge of triangles gives no clew to the forms and shapes which represent the beautiful. For the beautiful everywhere suggests or presents some traits peculiar to mind or the soul. Forms that can not be explained except by self-activity are the only forms that present the beautiful. Trigonometry, therefore, gives us the clew to the inorganic and what is mechanical, but art teaches us what is beautiful and reveals to us through it the spiritual.

Therefore, besides alphabets of touch, taste, smell, sound, and sight, as to inorganic forms, the senses should

have alphabets of beautiful forms of painting and sculpture, and of beautiful sounds of music.

One of the most important of all the writings of Herbart is the treatise translated and printed in Chapter VI (pp. 80 to 117) of the present volume, entitled "The Æsthetic Presentation of the Universe." It represents Herbart's voyage of discovery to find the most important determining element in education. He calls this master principle "morality," but in his analysis of it does not discriminate it from the science of the beautiful. He includes all ideals which *ought* to be realized, but which do not press upon us with external necessity—i. e., not *must* be, but only *ought to* be—under the name of æsthetic perceptions. Setting aside for the time being art and literature as perceptions of this character, he elevates into prominence the moral perception.

Herbart's insight into the importance of the moral leads him to subordinate mathematics and natural science to literature and the humanities—to all, in short, that gives a knowledge of human nature as an ethical principle for the conduct of life.

It is important, therefore, to consider this A B C of Sense-Perception in connection with his Æsthetic Presentation of the Universe, inasmuch as the charming work of measuring triangles may, in the hands of an enthusiastic teacher, be made to absorb too much of the youth's attention, and give his mind a too exclusive bent toward the mechanical view of the world, just as number work, if pursued in the elementary school to the point of making rapid and accurate accountants, will cramp the mind and arrest its development, retarding its progress into more humane studies. These trigonometric studies may have the same effect; they are, however, very excellent by way of a single device

if kept in strict subordination to that doctrine of the æsthetic presentation of the universe.

Herbart has given us this series of object lessons in trigonometry, and by the continued occupation in estimating and then actually measuring angles and sides of triangles the pupil will soon become able to forecast accurately the dimensions of the object which he sees. His mind will form a habit of unconsciously analyzing all objects which come before its perception into triangles. But by so much as the mind adopts this unconscious habit it will neglect to notice the elements of taste and gracefulness and the ethical or moral significance of what it perceives. But it is not a matter in which we can adopt one alphabet and totally neglect another. We must have all of these alphabets and yet have them in their due proportion: we should have A B C's for drawing, painting, and statuary; for botany and zoölogy; for literature and morals.

Herbart deserves the careful study of the teacher because of his painstaking investigation of branches of study in view of their value as materials of apperception, as materials by which to enable the mind to recognize and explain the objects of the world of experience.

W. T. Harris.

Washington, D. C., *May 28, 1896.*

PREFACE.

In the days of Horace Mann a wave of Pestalozzianism swept over the country. The uplifting effect of that period has not passed away. Its elevating results continue everywhere. The elementary education of the country having been placed upon a higher plane, on that plane still proceeds. Thousands who do honour to American elementary school-work are Pestalozzians, including under that name the disciples of Pestalozzi's disciple, Froebel.

We live in the beginnings of another educational reform—the Herbartian. It is not intended to supersede, but to enlarge and to fortify, Pestalozzianism. All thinkers on our national education feel that, in order to preserve its strength and to supply its defects, it must be apperceived, as Herbart would say, under higher points of view. The warmest friends and the keenest critics of our public schools are at one on this.

We have outgrown the limits of Petalozzianism just as our free-school system will no longer brook the restriction to the three R's. Many who know our public-school system best and love it most are persuaded that Herbartianism is the proper solution for the difficulties of to-day, as Horace Mann was rightly convinced that the doctrines of Herbart's predecessor, Pestalozzi, offered the true remedial agent for the difficulties at an earlier stage. It is hardly

possible to attend an educational gathering of fair pretensions to magnitude or dignity without hearing Herbart's name at least. It is almost impossible to take up a catalogue of teachers' books without noticing publications based mediately or immediately on his principles.

Already many teachers are guided by Herbart's ideas who never heard of him. Many subjects, from sympathy with the treatment given to others, are treated in a way more or less conformable to these ideas by writers not consciously belonging to any pedagogical school. American educators have begun to live, move, and have their being in an atmosphere of Herbartiansm. It is coming to be the pedagogic spirit of the times.

There is occasionally misconception so gross as to shake the belief that the person in error can have studied Herbart, though he may have studied in a fragmentary way Herbartian authors. There are superficialities and crudities, but not, relatively to the vastness of the subject, at all more than at the beginning of the manual-training movement.

Both the pervasive power and the prevailing defects of the movement seem to be due to the way of introducing it. Whether by design or accident, the American teacher has been made acquainted first with the General Pedagogy and the Psychology.

It was, perhaps, as good a way of beginning as any other. Had a commencement been made by introducing first the practical details, opposite and possibly greater defects would have been developed. There might have been—elder educators, at least, remember something of the sort in the movement for public kindergartens—local adoptions and narrowly practical adaptations. Whether this would have resulted in a greater number of well-qualified teachers is very much open to question. That it

would have failed to bring out the philosophical power of Herbartiansm is not so open.

Yet, after all, we need both a knowledge of principles and a knowledge of how to translate them into the practice of the class room. The Herbartian movement, it is conceivable, will be strengthened by placing in the hands of the thousands of teachers who do the actual class-room work a book strictly Herbartian and containing ideas applicable immediately in the daily work of education. For such a book the master has not left us without materials. To render these materials accessible to American teachers is, as a glance at the title-page and the table of contents will show, the aim of the present writer.

W. J. Eckoff.

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HERBART'S
A B C OF SENSE-PERCEPTION.

INTRODUCTION.

Historically, what is Herbartianism?—In the history of education a certain portion of mankind may at once be set aside as the Occidental fraction. It is loosely coincident with Europe and that western extension of Europe which we call America. The latter, though separated from the rest of Europe by broad seas as England is by "the narrow seas," is in language, jurisprudence, art, science, industries, religion, and education—north of Mexico as regards the origin of its population, also—European. In the history of education the United States is a European country like England, France, or Germany, and much more of a European country than either Russia or Turkey. The history of Occidental education commences with the Greeks.

For the present purpose we exclude from consideration all but intellectual education. This among the Greeks had two strands. The objective strand produced Thales, Archimedes, Euclid, Ptolemy, Pliny, Democritus; the literary, Sophocles, Demosthenes, Cicero, Quintilian. As above in the case of Europe, we here classify solely for educational history. Though the withering and narrowing brought upon the Greek education by the one-sided instinct of the Roman for conquering and governing had lapsed into its worst stage during the anarchy of the middle ages, the two strands remained nevertheless. Every study in the course

retained its Greek name — grammar, dialectics, rhetoric, music, arithmetic, geometry, astronomy.

To get back to the Greek level was the struggle of the Renaissance three hundred years ago, when the subsidence of mediæval anarchy allowed civilization to lift its head once more.

In the first struggle, unfortunately but unavoidably, the literary element was overemphasized. The reformers would admit nothing but Greek and such Latin as we have included in Greek civilization. Some would not accept any Latin word not found in the arch-Græcist, Cicero.

The reaction against this humanistic one-sidedness is the realistic pedagogy of Comenius's Orbis Pictus. It was the only schoolbook Goethe ever had. Goethe died in 1832. This shows that both sides held their own, as they did in Greek times and as they are doing to-day in the struggle between literature and science in our higher schools and colleges. The bilaterality of Occidental education extends through two and a half millenniums. Nobody need fear or hope much.

Meantime the non-scholastic world was wondering whether the battle of the pedagogues was worth disturbing one's self about. This scepticism of the non-scholastic world produced the most important experiment in all the history of education.

Ancient education ends and modern education begins, not where the break is usually made, but with John Locke. A great philosopher of the kind opposite to those who, to speak with Herbart, become great philosophers by forgetting their logic, he was well qualified for the Conduct of the Understanding of his pupil. Being a physician, continually compelled to practise the healing art, his attention was directed scientifically and sympathetically to the physical as well as to the mental nature of the child. Leading the life of a practical politician to the point of endangering his head, his opinion on the knowledge required for dealing

with men and things was not that of a pedant. Mingling on equal terms in the eminent and intelligent society of which his pupil's parents formed a part, and possessing their full confidence, he had in his power all that could enable him to form the one boy intrusted to his charge. Summing up all the conditions, it is impossible to see how they could have been more favourable for the success of the great experiment which was about to be performed. It succeeded perfectly. Locke took charge of his pupil, a weakly boy, educated him into a man physically well, and intellectually and morally fitted for his place in society; and, to crown this strangest experiment in human history, felicitously mated him in wedlock.

Locke, as Herbart has well observed, is the easiest philosopher to underestimate. His genius as an educator has never yet had justice done to it. His writings are almost ostentatiously negligent with the true Briton's contempt for fine points and studied self-advertisement. The Conduct of the Understanding and the Thoughts Concerning Education give scant intimation of his pedagogic genius. The doers of great historic deeds rarely possess the gift of telling them to advantage. Hence, the view here affirmed that educational history should divide into two main periods, and that Locke should commence the modern or naturalistic period, will seem to many paradoxical. The next paragraph will, perhaps, induce a different conviction.

The great deed of Locke was written up; it revolutionized Europe. Rousseau took up the whole experiment and put it into a novel—Don Quixote alone excepted—the most perennially fascinating novel ever written. Locke's pupil he called Émile. Instead of Locke he said I. He described the course that would make the boy physically robust and morally and mentally adequate to his place in society, and, to complete performing in print what Locke had performed in deed, Rousseau picked out the proper bride for his disciple, and in the end married him to Sophie.

The Anglo-Saxon Locke performed the greatest feat in educational history, but, with the curious idiosyncrasy of the English for belittling everything they do, almost failed to describe it. The French-Swiss Rousseau beautifully described what it is positively sure he could not do. To understand Locke we must read that Rousseauist masterpiece, Émile. It alone is worth the trouble of acquiring a reading knowledge of French. The question is occasionally asked why England and America should go to Germany for so much of their pedagogy. The answer is extremely simple.

Modern pedagogy was started by the Anglo-Saxon Locke as modern or, as the French have sometimes called it, romantic literature was started by the Anglo-Saxon Shakespeare. In both cases the Germans joined in, as they did at Waterloo, to help their Saxon kinsmen complete the victory.

Locke, who pleaded stanchly and victoriously the cause of Religious Toleration, and who anticipated also Rousseau's political writings in his Two Essays of Civil Government, is more than any other one the typical Anglo-Saxon philosopher. The spirit of religious and political freedom in his writings animated the patriots of the American Revolution. His Essays of Civil Government were the most widespread and thoroughly read political literature in the homes of the generation that fired the shot which was heard around the world. If the French wrote the Rights of Man, Locke anticipated them by establishing the rights of the pupil, just as the American Revolution, the child inspired by the spirit of Locke's writings, anticipated the French Revolution. The strong individualistic spirit of the Saxon race in religion and in politics is potent equally in the school. The free obedience to law which accompanies that spirit must pervade our education. No machine-drilled marionette, unthinkingly submissive to authority, is fit for American citizenship. On the other hand, no anarchistic tendency can be permitted to endanger its foundations. How to train to good citizenship in the only way possible—namely, by

intelligent self-determination to goodness—is the burning pedagogical question, for in it lies the future safety of the republic. That is conceded at all hands. In our day the pupil is once more the core of the question. State and Church are battling, or sometimes bargaining or compromising, over his possession.

To have placed the individual, the child, instead of the branches of instruction, into the forefront of pedagogic consideration is the great, the saving deed of Locke. He called the educators away from the wrong end of the telescope. There was at once a glorious view of the possibilities of instruction. That glorious view was ecstatically described by Rousseau.

But one thing remained to complete this naturalistic victory. It must be applied to mass education—the absorbing problem propounded, and scarcely more than propounded, by Comenius. The application was made by a lover of mankind, in whom devotion to his fellow-beings, and the intuition that arises from such devotion, amounted to genius—Pestalozzi. The French-Swiss Rousseau took the Saxon idea of education and converted it into a beautiful dream. The German-Swiss Pestalozzi converted the dream into a beautiful reality. Since Pestalozzi mass education is a fact. Occidental humanity has solved the great sphinx riddle which history has propounded to every race—how to perpetuate its civilization without petrifying it; how to pass it on to succeeding generations, not only without limiting it, but while positively enhancing it. Those who affirm that Occidental civilization must die as other civilizations have done are wrong. They were right before the Saxon conquest, which, as it won for Occidental civilization civic liberty, also won for it this larger hope, put into a dream by Rousseau, into reality by Pestalozzi. That is the reason—because it is so thoroughly Saxon an idea—that the Pestalozzian school system is the basis of American primary school work. Pestalozzi's boast, that finding the car of education going the wrong way he turned it and set it going in

the opposite direction, applies only to his accomplishment of the naturalistic idea for public school work. In the large aspect of the case the boast is sober truth as applied to the Saxon race in the person of Locke.

To state this truth is not to belittle Pestalozzi any more than it is to belittle him to say that he did not go to the root of the matter. This was reserved for Froebel, who, in the skilful adaptation of his means to his ends and in the profundity of his philosophy, possessed qualities to which Pestalozzi could not lay much claim.

When both the basis in method and mass education and the superstructure of the subject-matter were fully provided by the pedagogic toil of Occidental humanity from the Greeks to Pestalozzi, the time was ripe for a master mind to unify the result of the struggle by harmonizing it into a single system. Occidental education was moulded into a logical whole by Herbart, who was at once a profound philosopher and one of the most successful practical teachers upon record. That is Herbartianism, speaking historically. Now—

What is Herbartianism, philosophically?—Naturally, knowledge—and hence practice, dependent on it—begins with isolated facts picked up on occasion and utilized of necessity. Such knowledge, and its sometimes quite artful applications, are displayed by intelligent animals, savages, and the general mass of the population of nations. Even in the intellects which have attained the highest degree of culture, the discharge of a large number of comparatively complex functions is possible only through such knowledge. This is the empirical stage of knowledge, and, by implication, of the resultant art.

It is never wholly overcome. The man who should attempt wholly to overcome it would unfit himself for practical life. Empirical knowledge—it would be an improvement to revive the Baconian term knowledges—and the resultant empirical deftnesses retain their importance, in many lives comprising almost the whole and even in the

highest lives a large, though not necessarily the most important, part.

None the less, in all higher life empiricism is manifestly inadequate to lay hold of all the knowledge and to control all the functions. By contrast, repeated observations separate into classes, while by similarity they detect the presence of causality. This stage, wherein the mind centres on classification and causality, we term the inductive stage.

It is never entirely absent, even in a mind preponderately empirical, is especially pronounced in minds new to life, or to some phase of it, and is, in fact, the stage beyond which most of our scientific knowledge and of the arts dependent on it have not passed, nor for a long time to come are perceptibly likely to pass.

It is not, however, the highest stage of scientific development. This is attained only when a law, a principle, an axiom, is ascertained with such absolute accuracy that all the details that fall under it can be shown to flow from it of necessity. Being the highest stage, it is the latest and the most rarely reached. The consequent art partakes, of course, of the certainty of the science. When he is sure of his mathematical principles, the art of computation which a person bases upon it is in nowise inferior in the accuracy and certainty of its results. If there be any error the mistake is purely personal.

Let us illustrate these three steps from the history of astronomy. It was anciently possible both to Occidental and to Chinese civilization to predict eclipses. The art of prediction worked by rule of thumb. The knowledge was empirical. Kepler's laws of planetary motion were obtained by a combinatory toil of intelligent observation so prodigious as to be one of the best illustrations for Newton's definition of genius as patient industry. Still these laws remained inductive. It was only when Newton himself discovered the law of gravitation that our knowledge of astronomy became deductive.

We must again insist that a higher stage does not sweep

away the results of a lower. On the contrary, it renders them more available, both by organizing them and by eliminating mistakes. Our belief in Newtonian gravitation does not make us reject the laws of Kepler, but makes us see them far more plainly and necessarily as laws of Nature than the great discoverer himself did. And, again, belief neither in gravitation nor the laws of Kepler requires us to disbelieve astronomic facts empirically ascertained. On the contrary, by our higher point of view these facts reduce from chaos into luminous clearness and consistency. Incidentally to this clarification, mistakes are corrected. We no longer believe that planets have circular orbits, for Kepler's induction corrected the mistake of Copernicus by demonstrating that planetary orbits are elliptical. Kepler's correction, however, did not overthrow but fortified the Copernican hypothesis. Occasionally great men are inconsistent with themselves. A later stage of development, from a higher point of view, can rectify the inconsistency, though neither the great man nor his contemporaries could see it. And, to repeat it, as our knowledge rises to higher stages our practice approaches perfection. We now guide vessels around the earth by astronomy.

To apply these remarks and illustrations to pedagogic science and the art of teaching will return the answer to our question. Most teaching even to-day is done in an empirical fashion. This is decisively and incisively set forth by Spencer in discussing What Knowledge is of Most Worth? The perusal of that chapter may be recommended to any one whom personal observation does not furnish with sufficient evidence.

But as the educational life of the country became elevated into something better than that in the little red schoolhouse on the hill—venerable as that is as an historical memory—the crudity of the empirical teacher no longer sufficed. The common school teacher of the United States does not teach abstruse specialties. His success or failure is immediately gauged. By the survival of the fittest in the

struggle for official life a species of men and women is evolved who do not ask whether there is a science of pedagogy. A specialist in a higher institution, remote from the strong current of public opinion and engaged in a specialty which the lay mind can not fathom, may mask his comparative inefficiency as a teacher by his comparative eminence as a scientist so completely that not even his pupils shall know how poor an instructor he is. Consequently, he can afford to wonder whether there are such things as pedagogic laws. But for the common school teacher to contravene these laws and thus fail in his work is suicidal. He, accordingly, knows that there are pedagogic laws just as surely as there is a law of gravitation. He strives to learn them and to use them.

But the laws so learned are purely inductive laws. The great mass of our public school teachers are still in the Pestalozzian stage. They have learned to believe in the Kepler, so to speak, of pedagogy. And this is well, as far as it goes.

But the country is about to reach the last stage. There is an obvious drift toward unity. It is unnecessary to cite the Committee of Ten report as one of the indications. The result will be to unite into one system our schools from university to kindergarten, not with the external unity which legislatures and boards of education can decree, but with the internal unity of method and aim which can alone produce a living organism. The Newton in the case will be Herbart. It is not right to say that his system is the best system of pedagogy. It is so far the only one. Valuable empirical observations are to be found plentifully elsewhere. Many and important inductive laws we owe to others. But his is the only system. If it plays us false, as the Ptolemaic system did in astronomy, we shall have to begin all over again. But if it proves right, it makes pedagogy a systematic science, and our practice measurably certain. In addition, it will enhance the value of all previously discovered laws and facts by setting them in the right rela-

tions and correcting the errors of previous discoverers. Having considered Herbartianism philosophically and historically, we can be brief in our—

Description of the System.—In the first place, it makes the pupil the centre of education. It affirms the Occidental doctrine of individualism as wrought out in education by Locke and idealized by his successor Rousseau. But in taking this position does not exclude mass education like that of Pestalozzi in the primary school, nor mass education like that of Froebel for the infant. In the next place, it is for the first time in the history of education cogently logical in basing this individualism upon its scientific foundation, psychology.

The labours of Herbart in psychology are initiatory. In pedagogy they mark a summit. That ultimately this educator and psychologist is, like Locke, a philosopher, we have already stated. There are thus firm foundations for our Occidental pedagogy. It is not only the most congenial to Occidental civilization, but it alone rests on science; not, like Oriental education or, indeed, like Occidental education before it found this true expression of it progressive genius, on prejudice or immemorial prescription. Hence, it is not destined to perish as prejudices vanish in the progress of history, but to unfold with greater amplitude and lucidity as Occidental science and enlightenment progress. Its characteristic can be condensed into a watchword—Educative Instruction. Its cardinal affirmation is twofold. Education of man by man is impossible except through instruction. That instruction is valueless in the acquisition of which no education—or, in psychologic language, no apperception—occurs.

We start, then, from a basis of realistic—not, let the reader be absolutely sure to understand, materialistic—philosophy, and from the standpoint of the pupil's mind conceived of as a complex of psychic forces, acting, reacting, combining, and so mutually helping and thwarting each other. We find that a process of construction is neces-

sary, that, in fact, instruction to be educative must be construction. But in order that the presentations in the pupil's mind may enter into such constructions as will be at once clear as to details and systematic in their comprehensiveness, if we look at them in discussing the problem of intellectual education; or, expressed in terms of the problem of moral education, at once pure, strong, and definite in moral attachment and yet well balanced in justice of moral action; if, in other words, we would educate simultaneously the scientist and the man of character, it is necessary for us so to present the world as to make the whole of instruction —and, remember, we want no instruction that is not educative—an edifice constructed out of the sound raw materials of outer and inner experience; and we need so to conduct the process of construction as to engage the whole intellectual and moral nature of the student in it joyously.

This two-sided necessity Herbart has felicitously coupled by calling "the æsthetic presentation of the universe the chief office of education." If education will attend to this, its principal duty, we shall have not narrow specialists in science and morality, but right intellects and right characters; for, whatever the requirements of later life, the foundation was laid in the formative period of the student's own soul in "equilibrating interest."

But, again, in order to do so, we must take up into the student's mind equally the two great strands which have ever been present in Occidental education—the strand of knowledge and the strand of sympathy. Hence Herbart's insistence—so often misunderstood—for higher education in mathematics on the one hand and Greek literature on the other. That Herbartian mathematics is a very different thing from the ordinary dispensation of the text-books, and the Greek literature to be read by eight-year-old boys a very different thing from the corresponding line of educational publications current in our American high schools and colleges, goes without saying. As to the mathematics, the reader will see by and by.

These two chief elements of culture—knowledge and sympathy—must in each individual pass through the three stages which the race has traversed—the empirical, the inductive, and the philosophical in knowledge; and the empirical, the sociological, and the religio-æsthetic in sympathy. Enough, perhaps, has been said in these opening paragraphs of an outline to hint, at least, the all-comprehensiveness of Herbart's grand system. At all events, we must here break off, not only for lack of space, but lest from pedagogy we go off into psychology.

We wish to conclude by offering, and urgently, too, one essential caution. The only way to understand an author is to read him, not in an attitude of hostile, destructive criticism, but in a spirit of sympathetic, constructive appreciation. Fairness to the author requires it and the student's enlightened self-interest should enforce it upon himself as the most profitable method. Of course, no one is obliged to retain what in the crucible of experience and reflection he has found to be slags. Yet in the case of no thinker more than in Herbart's is it needful for the reader constantly to bear in mind the eminently sane advice of Goethe:

> "Gib dich an einen Meister hin;
> Mit ihm zu irren ist dir Gewinn."

We venture to translate it:

> "Unto a master yield thy thought;
> Erring with him thy gain is wrought."

PART I.

INTRODUCTORY WORKS.

CHAPTER I.

HERBART at the beginning of his career as a university teacher lectured on pedagogy to students in the University of Göttingen, in 1802–'03. The manuscript of the first lecture was not given to the world until after his death in 1841. The last four paragraphs here given constitute the beginning of a second lecture.

His reasons for eschewing both a definition and a history of pedagogy, as well as the succinct but cogent exposition of the insufficiency of empiricism as a substitute for pedagogic science, are noteworthy. His intimation, that the task of educating must not sacrifice the educator any more than those who are to be educated is also worth heeding. But far the most valuable portions are the discussion of tact, and the practically instructive parallel between tact and character. Without more introduction we give Herbart's

INTRODUCTORY LECTURE TO STUDENTS IN PEDAGOGY.

GENTLEMEN: You perhaps expect at the beginning of these lectures first of all a definition of my subject and next a eulogy, a history, or a synopsis of the science of pedagogy.

Only after a first attempt at separating the essential from the inessential can a definition be the significant

expression of the result of this entire inquiry. To one not having before him that which is to be separated, a definition shows neither what has been excluded nor the propriety of the exclusion. It comes as a surprise rather than as a support to one's thinking. Instead of a definition I shall from the crude idea of which we are reminded by the word education set forth the chief properties as far as necessary in order to start the threads of further investigation.

Just as little a eulogy! Such a crown might oppress rather than glorify the brow of my modest science.

It is, perhaps, appropriate to open with eulogies the exposition of sciences the propositions of which stand forth in ample definiteness, and the beneficent effects of which have been indubitably proved in general experience—sciences that have already reached the age of manhood. But the art of educating the youth of mankind is itself still a youthful art; it experiments, trains its powers, and hopes in the future for excellent accomplishments. But hitherto it is fain to confess that its trials have instructed it not so much what to do as what to avoid; that it still at every step dreads the superior power of interfering accident, which it would rather flee than combat; and that, as to its general principles, it still expects, to be sure, the dicta and objections of philosophy without, however, knowing whether, to begin with at least, it will by them be more instructed or more unsettled. Eulogies in the case of such a science can refer less to its actual accomplishments than to the hopes which we entertain as to its future. But even the foundations for these hopes are to be set forth only by the whole of these lectures. As I shall gradually develop before you the idea of our great art, and demonstrate more definitely the feasibility of carrying the idea into

effect, the respect which, I have no doubt, you have brought hither for pedagogy will heighten surely into confidence, possibly into reverence.

Nor shall I give you a history! What does the history of a science contain? Attempts, no doubt, made to construct that science. Who is able to estimate the value of such attempts and to perceive where in them there is progress or retrogradation? No doubt he who sees the best and shortest way which the attempts might have taken. Hence, as a rule, the history of an art does not become comprehensible and interesting till one has mastered the main ideas by which the attempts are to be judged. Then, in the case of mistaken measures, one can discriminate and esteem correct intentions, assign the proper measure to that which was missed by excess or weakness, and appropriately separate what is true and important from the insignificant, the erroneous, and the perilous.

Instead of a history of pedagogy there is great need, however, of your obtaining a distinct view of the existing condition of the art of education. To this end I recommend to you two means. In the first place, please to look back, each one into his own youth, and to recall not only how you yourself were educated, but also how you have seen others educated. In doing so it naturally is likely that not many of you will entirely avoid thinking of your teachers and educators either with partiality or disesteem. Your youth is, as yet, hardly far enough behind you for that impartial contemplation by which that portion of your history will become instructive experience. Especially, when one believes that one detects radical errors by which one has suffered, by which to a greater or less extent one has been irremediably warped, or at least irretrievably retarded, one has difficulty in not becoming unjust

and thankless, by forgetting how much the fault lay with the prevailing spirit of the time, how much one's education possibly rose above it, how many obstacles it had to contend with, how much worse one should be off without it.

But I admit that this does not exactly belong to the subject under consideration. Here it must be our aim to recognise faults as faults, however well they may be explained from circumstances. We must make it our business to free ourselves completely from the influence of habit, from force of which a father is inclined to repeat in his son the treatment he received from his father; to step, if possible, even beyond the limits of the present age as far as it might blind reason by authority; to take a full view, on the one hand, of the pure ideal, on the other of the existing means for realizing it, in order that we may at the least not miss the best that is possible in our very inception of the plan. Only in order to become acquainted with the means at hand, especially with those among them which pedagogy has already prepared for its use; in order the more surely to avoid the aberrations to which the age seduces easily, and against which on that account modern pedagogy cautions the loudest; in order to orientation by the nearest and for that very reason the most obvious experience, in a matter of experience such as education is—in order to all these things an attentive view of the present is needed. . . .

Neither shall I present to you a synopsis of my pedagogy. For would you understand me if, at this early stage, I were to speak of an instruction which at the same time is an education; of a broad division of the method of this instruction into synthetic and analytic; of an æsthetic presentation of the universe as the ideal of education? In my theses I have reluctantly declared mathe-

matics and poetry to be the chief forces in education, and I barely now risk intimating to you so much of my view of the whole of the pedagogic task as is done in saying that I hold the culture of the imagination and of the character to be the extremes between which it is comprised. Paradoxes are not the best means for preparing the mind for a proposed investigation.

I hope to contribute a little more toward such a preparation by a preliminary explanation of the way in which I intend to treat my subject.

Discriminate, in the first place, between pedagogy as a science and the art of education. What is the content of a science? An orderly combination of propositions, logically constituting a whole and where possible proceeding one from another—corollaries from fundamental principles, and fundamental principles from axioms. What is an art? A sum of skilful devices and methods which must be combined in order to secure a certain purpose. Science, therefore, demands the derivation of propositions from their logical grounds—philosophic thinking. Art demands a constant activity in conformity with the mere results of science. An art while it is being exercised must not become lost in speculation. Its aid is invoked by the instant moment. Its resistance is called in against a thousand hostile accidents.

Furthermore, discriminate the art of the expert educator from an isolated application of educational art. Knowing how to deal with all types of mind and all stages of growth makes up the former. The latter may be successful by accident, by sympathy, by parental love.

Which of these three cycles is the cycle that we are considering? Obviously, we lack opportunity for actual practice, and, still more, we lack the opportunity for those manifold exercises and experiments by which alone the

art can be acquired. Our sphere is that of the science. Now I must beg that you will consider the relation between theory and practice.

Theory in its universality stretches over an expanse of which any one in his practice touches on but an infinitely minute part. On the other hand, in its indefiniteness, which is the immediate consequence of its universality, it passes by all details, all the individual circumstances that surround the practical teacher at every given moment, and all the individual measures, reflections, and exertions by which he must respond to those circumstances. In the school of science, therefore, we shall learn both too much and too little for practice. This is the reason why all practical workers have habitually a strong dislike to entering, in respect to their arts, into rigid, thoroughly investigated theory. They very much prefer throwing into the balance against the latter the weight of their experiences and observations. On the other hand, it has often and prolixly to the point of fatigue been proved, set forth, and repeated, that mere practice produces strictly only routine and extremely limited and wholly indecisive experience, the contention being that we must learn from theory how to question nature by experiment and observation ere we can draw from her definite answers. Of pedagogical practice this contention is true in the amplest measure. In it the activity of the educator goes on without cessation. Even against his will he acts well or badly, or at the least neglects to effect that which might have been effected. Quite so, without cessation, the reaction, the result of his activity, returns upon him. But it does so without showing him what would have happened had the action been different; what the result would have been if he had proceeded with greater power and wisdom; if he had been master of pedagogic means, whose very

possibility he perhaps did not dream of. Of all these things his experience knows naught. He experiences only his own self, only his own relation to men, only the miscarriage of his own plans, without discovery of cardinal faults; only the success of his own method, without comparison with the possibly much more rapid and elevating progress by better methods. Thus it may happen that a gray-headed schoolmaster to the end of his days, yes, that a whole generation or even series of generations of teachers, ever proceeding beside and after each other on the identical or on scarcely deviating tracks, suspect naught of what some young beginner experiences the first hour, at once and with absolute decisiveness, by some lucky hit or by some correctly calculated experiment. Nay, this not only may occur, it does occur. Every nation has its national circle, and, with still greater definiteness, every age has its time circle within which the pedagogue as well as every other individual, with all his ideas, inventions, experiments and resultant experiences, is inclosed. Other ages experience something else, because they do something else. It is everlastingly true that any sphere of experience without an *a priori* principle not only has no right to speak of absolute completeness, but that it can not even approximately state its degree of approach to such completeness. It is for this reason that a person without philosophy so easily imagines himself to have made far-reaching reforms in education when he has only improved some trifle as to the way of doing things. Nowhere is the philosophic circumspection given by general ideas so needful as here, where daily action and the multiplicatively impressive experience are so powerful to contract the circle of vision.

But in every theorist, no matter how good a one he may be, if he practises his theory, and provided only that

he do not proceed with the cases occurring in his practice with pedantical slowness, like a little boy with a sum in arithmetic, there inserts itself quite involuntarily a link intermediate between theory and practice. There is, to wit, a certain tact, a quick judgment and decision, not proceeding like routine, eternally uniform, but, on the other hand, unable to boast, as an absolutely thoroughgoing theory should, that while retaining strict consistency with the rule, it at the same time answers the true requirements of the individual case. Exactly because such a recollection, such a complete application of scientific propositions, would require a supernatural being, there inevitably originates in man as he is, out of continued practice, a mode of action which depends on his feeling and only remotely on his conviction—a mode of action rather giving vent to his inner movement, expressing how he has been affected from without, and exhibiting his emotional state, than the resultant of his thinking. "But what sort of an educator is that," you will say, "who depends on his whims and abandons himself to the pleasure or displeasure caused in him by his pupils!" And what sort of an educator, I ask, is he who would praise his pupils heartlessly and censure them by the book, ratiocinating and calculating while the boys are committing folly after folly, and incapable of opposing the energy of a swift and manly will to these often very forceful natures? Let the question and counter-question balance, in order that we may return to our assertion that inevitably tact occupies the place that theory leaves vacant, and so becomes the immediate director of our practice. Supposing the theory true, happy he, no doubt, in whom this regent is a truly obedient servant of the theory. For the question on which depends a man's being a good or bad educator is solely this—how tact forms itself

in him, so as to be faithful or so as to be false to the laws enunciated by pedagogic science in its universality?

Let us reflect somewhat further as to the effective causes, as to the influences, on which depends the manner in which that educational tact becomes ingrained in us. It is only formed during practice, and by the action of our practical experiences upon our feelings. This action will result differently as we are differently attuned. On this, our mental attuning, we can and should act by reflection. It depends upon the correctness and weight of this reflection, upon the interest and moral willingness with which we give ourselves up to it, whether and how before entering upon the office of education and, whether and how, consequently, during the exercise of that office, our mental tone will order our mode of feeling, and finally, together with the latter, will guide the employment of that tact upon which rests success or failure in pedagogical endeavour. In other words, by reflection, reasoning, inquiry, in short, by science, the educator must prepare not his future action in individual cases so much as himself, his tone of mind, his head as well as his heart, for correctly receiving, apperceiving, feeling, and judging the phenomena awaiting him and the situation in which he may be placed. If he has anticipatingly indulged in extensive plans, the practical circumstances will mock him. But if he has equipped himself by fundamental theories, his experiences will be plain to him and teach him what is to be done in every case. If he does not know how to distinguish what is significant from the insignificant, he will fail to attend to things vitally necessary and wear himself out on what is useless. If he confounds a lack of education with feebleness of mind, and crudeness with alignity, his pupils will startle and bewilder him every my. If, on the contrary, he knows the essentials upon

which his work depends and the fundamental traits of good and evil disposition in the youthful mind, he will know how to grant to himself and his pupils all the liberty necessary for cheerfulness, without on that account neglecting duties, loosening discipline, and opening a free road to vice and folly.

There is then—this is my conclusion—a preparation for the art by means of the study of science, a preparation of both the understanding and the heart before entering upon our duties, by virtue of which the experience which we can obtain only in the work itself will become instructive to us. Only in action do we learn the art and acquire tact, aptness, quickness, dexterity; but even in action only he learns the art who has in previous thinking learned the science; has made it his own; by it has attuned himself; has predetermined the impressions to be made upon him by future experience.

Therefore one must not at all expect of theoretical preparation that out of its hands he will go forth an infallible master of the educator's art. One must not even demand of it the special instructions as to procedure. One must have faith that he will have enough invention to hit upon the particular thing needful to be done at any instant. One must expect instruction even from the mistakes he is going to make, and one may do this in pedagogy rather than in a thousand other occupations, because here, as a rule, every single action of the educator taken by itself alone is trivial, infinitely more importance attaching to the procedure as a whole. One must tax not even one's memory to carry constantly the innumerable details which will require to be observed.

But, on the other hand, one must fill one's mind with the considerations which concern the dignity, the importance, and the main auxiliaries of education. Let

there ever hover before the educator's mind the picture of a pure youthful soul which under the influence of a moderate happiness and tender love, under many a stimulation of the mind and many an appeal for future action, develops uninterruptedly and vigorously, with ever accelerating progress. Let him in the beginning abandon himself to his imagination and adorn the picture with all that can charm; but in the next place, let him call to aid the strictest critical reflection, to point out to him what in his picture is arbitrary poesy, baseless reverie, without connection and consistency—what, on the other hand, was the demand of reason, the essential quality of the ideal. Having now framed his concept of a boy, not such as he should be pleased to educate, but such as would be truly worthy of an excellent education, let him frame in thought a teacher fitted for that boy, and again, not so much the companion of every step, as Rousseau does, not the warden, the slave chained to the boy, whom he and who him deprives of liberty, but the wise leader from afar, who by profoundly penetrative words and strength of conduct at the right time knows how to make sure of his pupil, and then dare calmly leave him to his own development in the midst of play, and contest with his mates; to his own aspiration to the activity and honour of men; to his own revulsion at the examples of vice by which the world, according to our choice, seduces or cautions.

Let us seek rather to fathom and divine the words and conduct of such a guide from afar. For, if it be impossible that as much time as a friend of youth willingly dedicates to it can suffice for education, then education itself is impossible; since if he is to give up to it all his hours, if he is to give to it wholly his best years—a claim so often made upon him—or even if he is but to

sacrifice to it their best part, he must neglect himself, and the relation between educator and pupil becomes a ceaseless, unnatural strain, which consumes the educative force itself. This is to give to youth overseers, not true educators. Our science must teach us an art which above all continues the education of the educator to a high degree. This art must act, moreover, with such concentration and intensity, with such accuracy and sureness, as not to be obliged to assist the student every moment, being able to despise and neglect the larger part of accidents; nay, if need be, even utilizing for its work important interferences of fate. For fate, circumstances, the education of the outside world, of which pedagogues are wont to complain so loudly, do not always influence the student unfavourably, and almost never do they so influence him in every respect. Education itself, once it has gained a certain degree of power, is able very often to turn those influences in the direction of its purposes. Even as it is, the world and Nature, take them as a whole, do much more for the pupil than education can, upon an average, pride itself on doing.

I have now described sufficiently, I presume, what is my intention of the science which I wish to teach. How far I fail of my goal, only he who attains it can measure. But to lead you nearer to it than you, at any rate, would have come, is the merit I should like to acquire.

It only remains to add something on the peculiar nature of this science in order to derive from it my suggestions for making the best use of this course of lectures.

You see from the preceding that my attempt will be in the direction of developing and of vivifying in you a certain pedagogic disposition, which must be the result of certain ideas and convictions concerning the nature and educability of man. These ideas I shall be obliged to

adduce, to justify, and then to connect, construct, and fuse together, so as to have them produce that disposition and so as to enable the latter to bring about the pedagogic tact which I have described. But to adduce, justify, and construct ideas is a philosophic business of the noblest, but also the most difficult kind—the more difficult here because I can not presuppose the purely philosophical basis on which I ought to build, especially not the psychology and ethics. How I shall go about to render comprehensible to you the results of my speculation without setting forth the speculation itself I can describe only approximately thus: I shall appeal to your knowledge of human nature, especially to your self-observation, in which the results of correct speculation must occur, although as yet in a dim, crude, and indefinite condition. But especially I beg you to have patience if my main ideas are composed but slowly out of their elements, and if I am under the necessity of forcing my way through all sorts of obstructive bramble. After all, everything will depend upon the finally resultant clearness and certainty, upon the energy, upon the impressiveness with which results infix themselves and prove their effectiveness in yourselves. In this respect, to be sure, very much will also depend upon how thoroughly you have mastered those sciences and exercises in which we shall recognise the most important auxiliaries of education. Among them I reckon especially Greek literature and mathematics.

To lead us back to the ideas of the previous lecture, take an illustration. Conceive of a man of character—of moral character, if you please—only do not think merely of what is called a good, honest, law-abiding man, but hold present to your minds a man in whom the moral element has grown into that decision, steadiness, and

swiftness of execution which with especial propriety deserves the name of character. What impels the man to action? A moral system neatly written up and deposited in his memory, in which he looks up the proper rule, as in a lexicon, or, to make a more appropriate comparison, as a judge does in turning to a statute book? Is it not rather a tone of mind, simple, strong, and never more to be effaced, which has resulted from his assigning, by a long, attentive, and impartial contemplation of human relations to himself and to all that surrounds him, the place appropriate to each, he now, carrying with him everywhere the feeling of universal order, being inclined to note at once, to measure off involuntarily, where and how much the established order has been infringed, and immediately following out the consequent impulse to toil, to be unable to rest, until he has done what in him lies toward the recovery of the moral order and its better future confirmation? Thus what he does is but the infallible reaction against the impulses which he receives. His actions are determined infallibly by the peculiar and especial way in which he, by virtue of his feeling for moral order that has resulted from his judgment of human relations, is struck and incited to action by the incidents which occur about him.

Here you will recognise again the link intermediate between theory and practice, of which I spoke yesterday—tact, that mode of decision and judgment converted rather into manner and morals than determined by rules distinctly thought of. This tact, which impels the man of character to swift and resolute action, is especially needed also by the educator, in order that he may know on the spot what is to be done and do it rightly and with energy. If an educator lack this tact his personality will not have weight; never will he prevail by authority;

never, as he certainly should, enforce by his mere presence that discipline by which the impetuosity of boys is broken and led back to order so much more advantageously and surely than by all coercive measures. But as there are not only moral, but very many species of characters, so also there are very many species of tact, manners, and ways among educators. Not decision, not swiftness alone, constitutes excellence. As there is a schooling for moral character, so also is there a schooling for pedagogic tact. And in these schools there are sciences; there is an ethics; there is a pedagogy. Both, provided they know their objects, will work by their presentations in such a direction as to generate not many isolated rules but some main convictions in the minds, and to strengthen, confirm, and exalt them into living enthusiasm; to wit, those convictions which are capable of securing to the moral or the pedagogic tact to be acquired its true direction in the future.

I have, then, myself prescribed the direction which in these lectures I should give to my exertions for the good cause. Let me have the assistance of your attention. Let me have, when I am mistaken, even the assistance of your doubt, and the straightforward and emphatic communication of your objections, in order that co-operating we may render to the education of mankind good service. Let it be far from us to bring still more confusion into that holy enterprise.

Taking now in hand the concept of education in order step by step to discriminate its principal elements and most important presuppositions, as well as the demands it makes upon us, let us contemplate, in the first place, the subject upon whom all education must be directed. This subject is without any doubt not only man in the most general sense, but man considered as a variable being; as

a being—that is, making transitions from one state into another, while yet capable of remaining with a certain degree of persistence in his new condition.

The manuscript ends with the words "Two properties." These words begin a new sentence. This would seem to indicate that Herbart proceeded next to explain his theory, as he does at large in his more elaborate works, that the educability, intellectual and moral, of human beings presupposes two conditions: First, our representations or mental pictures are modifiable; second, our representations determine our actions.

CHAPTER II.

THE firmness and breadth with which Herbart treated pedagogy was the result of thorough preparation. When at the age of seventeen, being head of the incoming senior class, he delivered the farewell address to the graduates of the *gymnasium* of his native city of Oldenburg, it proved a good enough production to be solicited by Herr von Halem for publication in the Oldenburg Miscellany, and the Latin valedictory, when he graduated in the spring of the following year, 1794, was not inferior. The former paper treated the General Causes of Growth and Decay in National Morality. The latter instituted a comparison between the thoughts of Cicero and Kant on the *Summum Bonum* and the standard principle in practical—or to use the less desirable, because less practical, English term—in moral, philosophy.

It is evident that he had even at that early age specialized on ethical investigations. From the *gymnasium* he went to the university at Jena, where he came into close contact with his "great teacher, Fichte," a contact by no means always tamely receptive on Herbart's part. Being especially strong in Greek and mathematics, an interested

experimenter in science, especially in chemistry, and a lover and composer of music, the "many-sided pedagogue," equally removed from the pedant and the sciolist, but prepared by the study of ethics and philosophy for his task, found the one training needed to complete him. A patrician Swiss family engaged him as tutor for its three sons. It was, with great modifications, the experience of Locke over again. Herbart's scholastic, philosophic, scientific, and ethical preparations, the need of adapting them all to the varying requirements of pupils of three different ages yet simultaneously to be instructed, and the possibility and necessity of individualizing in order to meet the expectations of a critically sympathetic family, produced the first germs of that system of education which we call to-day Herbartianism.

To the end of his days Herbart bore evidence of the psychologizing effect of his tutoring period. In his later years, and with ample experience in a larger field, he never swerved from the conviction that to make a profound pedagogue, conditions such as those under which he elaborated the elements of his system were ideal. The public school teacher dealing with masses is precluded from individualizing, and, therefore, from close and accurate psychology; he can at best only psychologize in the rough. On the other hand, it is quite conceivable that Herbart's system might have failed of applicability to mass education had he not providentially met during this formative period the great educator who was at that time reducing the naturalistic system of Locke, or of Rousseau, if one prefers, to public school practice.

The first meeting between Herbart and Pestalozzi, who was thirty years his senior, took place in 1797. A visit to Pestalozzi's school, and an abiding interest in Pestalozzi's subsequent publications, made Herbart a Pestalozzian, but a good deal more than a Pestalozzian—to wit, a man who was able to assign to the work of "the noble Swiss" the province within the entire realm of education, where it must be of permanent value, and at the same time able to define the limitations of that value.

After his three years' tutorship in Switzerland he returned, not indeed to his native grand duchy of Oldenburg, but to the neighbouring republic of Bremen, residing at the home of his fellow-student, the later eminent burgomaster Smidt, to whom Herbart's main work in education, the General Pedagogy, is dedicated.

Here, while preparing for the university tutorship, of which we have just presented a specimen, he was privileged with the society of three ladies of culture and wealth, related with more or less immediacy to his friend, and forming the nucleus of a group of mothers warmly interested in the study of pedagogy. To these ladies the work we are about to communicate is dedicated. It made its first appearance in 1802, in the January number of the Irene, a periodical edited by Herbart's old fatherly friend, von Halem. In transmitting the manuscript Herbart remarked: "In fact, a second essay is needed to expound the subject beyond the necessary limits of the Pestalozzian view. This counterpart should present Æsthetic Perception as the nerve of education. I have dropped a hint concerning it at the end of the paper." The reader will find this "counterpart" on Æsthetic Perception in Chapter VI of this part.

Now, as to the use to be made of the article about to be presented. It is twofold. In the first place, it shows the relation between Pestalozzianism and Herbartianism. That is an essential advantage to an American reader, the bulk of our common schools being still on the Pestalozzian basis, while the best are enlarging to that wider scope and assuming that higher point of view for which Herbartianism—a term which includes, not excludes, Pestalozzianism—stands.

That this enlargement and elevation of our public education to correspond with the ceaseless enlargement and elevation of our national life will continue, is certain. Now, no other work in pedagogic literature gives in so few pages so substantially correct an idea of the inner nature of the change through which we are passing. But no one can grasp the idea who has not really understood our American

public school system. The man who believes it to consist of resolutions by school boards and rulings by superintendents, and devices gotten from articles in school journals, instead of realizing that all these and kindred agencies are but the outward and visible signs of an educational theory extending from Russia to Japan, and taking in France and America in its course, and that that theory is Pestalozzianism, has not yet begun to understand the moving spirit of our common school work. That man may have Horace Mann in effigy over his desk and Barnard's American Journal of Education on his shelves, but he has understood the historical connection of neither.

If the reader of the present pages is consciously in that position, the following article may be of a second fundamental use to him. By reading and weighing it—for Herbart writes compactly, and every paragraph contains matter for reflection—with especial emphasis on the sections descriptive of Pestalozzianism, and by comparison with what he knows of our public schools, he may be able to see whence we get a number of practices that he has perhaps attributed to the ingenuity or the incapacity of school teachers and school journals.

But to supplement this conception to any serviceable degree he should re-read Spencer's Education, at least the chapters on What Knowledge is of Most Worth? and on Intellectual Education. The critics of Spencer are congeners of the critics of our public schools. The reason is patent enough to one familiar with pedagogic history, though apparently but little recognised by critics and defenders of Spencer and our common schools.

Both Spencer's Education and our public schools are emanations from Pestalozzianism. Perhaps the careful reading and pondering of the following article will at least suggest how the public schools will be improved by taking up the Pestalozzian views that animate them into a wider or deeper thought resulting from a higher point of view.

On Pestalozzi's most Recent Publication: How Gertrude Taught her Children.

A Communication Addressed to Three Ladies.

The long-expected book has come into our hands. Shall you find confirmed or disappointed the beautiful faith with which you interpreted what I was able to tell you about Pestalozzi and his enterprise? You note already one deficiency; the book does not read easily enough. Will you allow me to make the attempt to place you at once at the middle of the subject? If I succeed, Pestalozzi's inequalities of presentation will retard you but little. In any event, I know, you do not judge the cause by the style of its expression. You do not reproach a man of sixty for desiring to do no more than communicate with us hastily. You find it natural that he, being full of bitter pain at the sufferings of his people, and pushing his way down into the lowermost class as though driven by the enthusiasm of joy and the fire of youth, in order to teach little children their letters, should pour forth words of power, although, to be sure, a cool, precise description of his experiments would have been to us more welcome and more instructive.

You know that I saw him in his schoolroom. Permit me once more to call up the scene. A dozen children from five to eight years of age were called to school at an unusual hour in the evening. I feared I should find them out of humour, and that the experiment which I had come to see would fail. But the children came without a trace of repugnance; a vivid activity continued uniformly till the end. I listened to the noise of the whole school speaking in concert, or rather, not a noise, it was a unison of words, extremely audible, like a chorus keeping time, and as powerful as a chorus, as firmly bound, ad-

hering as definitely to that which was being learned, so that I had almost to make an effort to prevent myself from being changed from an inspector and observer into one of the chorus of learning children. I went about behind them in order to hear whether there might not be some speaking either not at all or carelessly. I did not find one. The enunciation of those children was pleasing to my ear, though their teacher had himself the most inarticulate enunciation, and their tongues could not, surely, have been educated by their Swiss parents. But this was easily explained. Keeping time and speaking in concert carries with it a pure articulation. No syllables can be slurred over. There is time for every letter. Thus the child forms its enunciation by constantly speaking aloud with the natural strength of voice. Neither was their universal and persistent attention a riddle to me. Every child's mouth and hands were simultaneously occupied. On none was the yoke of inactivity and silence imposed. The craving for diversion was satisfied. Natural vivacity did not require any outlet, and the current of concert speaking allowed none.

I was greatly pleased with the clever use of the transparent horn leaves with inscribed letters, which, while the children were memorizing, moved about in their hands continually, and being a mute but dexterous writing-teacher, when placed over the tracings made by their slate pencils detected the faults instantaneously, and made an appeal to them to do better. Even now, whenever in my mathematical occupations I place figures upon the blackboard, I rebuke my hand for its inability to draw as firm straight lines, as correct perpendiculars, as exactly round circles, as did these six-year-old children. But far more than on account of the accomplishment they have acquired, I deem them fortunate on account of the steadi-

ness and energy of mind which they have gained by holding on without swerving to the idea of rounding until the intently aiming eye and obeying hand have completed the circle, quite slowly and surely, in one faultless drawing.

But why did Pestalozzi cause so much to be memorized? Why did he seem to have chosen the subjects of instruction so little in accordance with the natural inclinations of children? Why did he make them always study or practise? Why never converse with them—never chat, never joke, never tell a story? Why were the sentences so disconnected? Why did the names stand isolated by themselves? Why was the whole range of devices for softening the rigidity of school life despised here? In all other respects Pestalozzi is at first sight a man full of love and friendliness. He greets so humanly everything human. His first word seems to say to you, "Whoever deserves to find a heart, finds one here." Why did he not pour forth more joy among the children who filled his whole soul? Why did he not combine more of the agreeable with the useful?

These questions did as a fact not perplex me as much as they might, perhaps, have shaken the faith of others. I was prepared by my own experience and experiments to estimate the mental powers of children very much more highly than is usual, and to look for the cause of children's pleasure or displeasure at instruction elsewhere altogether than in superfluous dallying on the one hand, or the supposed dryness and difficulty of things demanding seriousness and attention on the other. What is deemed by the teacher the easier and what is deemed the more difficult, I had several times found in children strikingly reversed. I had long held their feeling of a clear apperception to be the sole and genuine spice of instruction, and a regularity of sequence perfect and adequate in all respects

was to me the grand ideal in which I saw the thorough-going means for securing to all instruction its rightful effect. The main endeavour of Pestalozzi, as I was given to understand, was exactly the same; namely, to find this sequence, this arrangement and combination of all things which must be taught either simultaneously or successively. On the supposition that he had found it, or at least that he was on the right way thither, every inessential addition, every adventitious aid would be an injury. It would be reprehensible, because it would distract attention from the main point. If he has not found that sequence, it still remains to be found, or at least to be amended and continued. But even in that case his method is correct; at least to the extent of throwing out the injurious additions. Its laconic brevity is its essential merit. Not a useless word is heard in his school; the train of apperception is never interrupted. The teacher pronounces for the children constantly. Every faulty letter is expunged from the slate immediately. The child never dwells on its mistakes. The right track is never departed from; hence every moment marks progress.

But the memorizing of names, of sentences, of definitions, and the seeming carelessness whether all this was understood, made me doubt and caused me to inquire. Pestalozzi answered me by a counter-question: "If the children did not think in doing it, would they learn so swiftly and cheerfully?" I had seen the cheerfulness. I had no explanation for it, unless I assumed that it was accompanied by inner activity. Continuing the conversation, however, Pestalozzi led me to the idea that, after all, the intrinsic comprehensibleness of the instruction is a matter of far greater importance than that the child should understand on the instant what is taught at that instant. Most of what was memorized related to subjects

of the children's daily sense-perceptions. The child bearing a description in the mind left the school, met with the object, and though it did not comprehend the sense of the words until now, did comprehend it more perfectly than if the teacher had attempted to explain his words by other words. The happy moments of comprehension, and especially those of deeper pondering and connection, in short, of reflection, do not fall exactly within determinate lesson periods. Let the lesson give what is comprehensible and set together that which belongs together. Time and opportunity will afterward supply the concept and will correlate what was set forth together.

In connection with this, we must not forget that we are speaking only of small children. To them, a word, a name is not as to us merely the sign of a thing. The word itself is a thing. They linger upon the sound. Not until the latter has become commonplace to them do they learn to forget it in attention to the thing itself. You often hear a child for fun pronouncing one and the same word with all kinds of modifications. It plays with the sound. It is occupied altogether with the difference between two similar tones. It is to be presumed that the child is so occupied while reading Pestalozzi's alphabetical register of names, where a word changes, but gradually, into another. This is the only plea I can make for this alphabetical order. After all, however, I should restrict its use to the first, merely preliminary, acquaintance with names.

Thus far I have sought to entertain you with such externals as, perhaps, strike one most immediately. Let us now penetrate more profoundly into the core of the subject.

This core, I must beg you to bear in mind, is not the core of your cares as mothers and of your immediate wishes. The welfare of the people is Pestalozzi's aim—

the welfare of the common, crude population. He desired to take care of those of whom fewest do take care. He did not seek the wreath of merit in your mansions, but in their hovels. Though he be able occasionally to bestow upon you a useful piece of advice, that is a secondary matter.

I know to whom I am writing. This difference from you is not repugnant to you. Your interest extends with equal ease and joyousness as far as the furthest limits to which the activity of such a man can possibly penetrate or strive to penetrate. And it is necessary that this disposition should remain ever present with you while you study his work. Otherwise, you would be unable to recognise the adequacy of his procedure, and you would be just as unable to determine correctly the application that you have to make of it for yourselves. In the mirror of individual cares everything might easily appear distorted. You would find the whole too rough and clumsy in design, the method too stiff and tasteless. The most important part of education, the gentle nurture of the heart, you would find lacking absolutely.

Pestalozzi speaks of the children of beggars. He says he understands the ideal education for them as comprehending tillage, manufacture, and commerce. His devices are intended to serve them at first instead of their common, miserable instruction at school. It is his intention to place in the hands of wholly ignorant teachers and parents such writings as they need only to cause the children to read off and learn by heart, without adding anything of their own. What he believed could be carried into effect most immediately he preferred; he must have his levers sturdy enough not to break even in clumsy hands. The book in which, under the form of letters to a friend, he describes the outlines of such a plan, belongs

really in the hands of such men as have influence on the organization of the lowest schools and upon parents of the lowest social ranks. Such men would be able to spread his actual schoolbooks, which are to be published in the future. What is faulty in the whole publication therefore is, perhaps, its title, which brings it immediately into the hands of women, of mothers.

Perhaps it hardly now seems conceivable to you that such a method should be invented for your benefit also. Let us look into it. Invariably the most pressing needs are the most universal needs also. It is therefore certain that one who is endeavouring to provide what is most urgent for all is also caring for us.

In instruction, what is the most urgent element? Where does it lie in the realm of all that can be taught and learned?

Should you say it is a little of everything? A little natural history, a very little geography, a few anecdotes from history, a few little anecdotes of noble characters, great men and well-behaved children, now and then an Æsopian fable, some lucid exercises in the use of grammatical cases, a few names of stars, ancient gods, and chemical preparations, now and then a conundrum, a *bon mot*, or an example in arithmetic—excuse me from continuing the list. That would be very convenient for Pestalozzi; it would enable him to save the great pains of finding out the right sequence in instruction. On the contrary, he would only need to shake up well so checkered a store in order to provide much variety and avoid fatiguing by monotony. The sequence of subjects would be a matter altogether of indifference. There would be no antecedent and no consequent; no one thing to presuppose any other. Accepting this arrangement then, in the memory as well as in the understanding of the child, each bit of

instruction will easily give place to another. What its little imagination finds piquant it will for a few weeks tell to its aunts and uncles. It will be applauded for a few odd combinations of its own. Upon occasion of the first truly interesting occurrence in its own experience it will forget the whole worthless mass. On this subject a good deal has been said and a good deal more might be, which is excluded by considerations of space.

Would we really find the most urgent business of instruction it is likely that we must search somewhat more diligently; it is improbable that mere guesswork will disclose it. I therefore beg for a prolongation of your patience. I fear almost that Pestalozzi, by betraying it too quickly to readers still requiring to be instructed by him, failed to make it lucidly convincing to them.

Without doubt the most necessary instruction must be that which teaches man what he most needs to know. Now, what is needful to us is needful either to our physical or our moral nature. We need it either as sensuous beings to enable us to live or we need it as beings in the social relations of citizenship, family life, and so forth, in order that we may know and do our duty.

Agriculture, manufacturing, commerce, and all other gainful art and science pertain in the first class; religion, ethics, notions of civic rights and obligations belong to the second.

Really, everybody not intending to be a sloth and a glutton, a being without duty and without right, needs instruction in both classes.

But trades and arts, as well as relations which impose duties upon us, are nowadays so composite that instruction in them of necessity must be composite also. Thus it is complicated in a number of ways, which constitute many simpler species of instruction.

The school is not the place where it would be possible to educate a man wholly, or even mainly, either in the arts or in respect to ethics. Every one is obliged to learn his trade with a master in that trade, and his moral nature man in truth only forms for himself and in the midst of life. The school, therefore, can undertake a part only of the instruction which a man needs. It subdivides and therefore facilitates his learning. It can guide boyhood to relieve youth of a part of the toil.

To a youth all the parts of his business are equally necessary; he must learn the whole. In order that this whole be not too great the boy should anticipatively comprehend as much as he can comprehend. Now, this much should not be many segregated bits of knowledge, isolated turns of practical cleverness, disconnected moral acts. In order to master a large number of details the boy would have to be numerously endowed with powers. On the contrary, the most general information, the abilities whose influence extends the furthest, the aptitudes that break the road to the furthest extent for the whole of education, the anticipative training which finds application in the greatest number of moments in life, and which in every new application bears new fruit—in a word, that which afterward makes for the greatest number of possibilities is that which merits being first in order, so as to make all the rest possible, so as to enable the boy to make the most of life.

Let us rapidly review this! Of what is needful, we said the school can furnish something, but not everything. The school is to do as much as it can do. For its purposes, therefore, the first and most important are those means of education the efficacy of which extends furthest, begins earliest, and is most frequently renewed by opportunity. The school prefers what is most general, because

it facilitates the greatest number of subsequent proceedures.

For he who knows the general already knows something of each particular in which the general occurs. He is prepared to perfect his learning of the particular. He feels impelled to extend the knowledge, the acquisition of which he has begun. The difficulties, being already half conquered, do not make him hesitate much. Time and desire will sooner suffice. His attention is won for every object in which he finds bound up together that which is known and that which is new to him. An open entrance into some mysterious darkness entices to enter and investigate.

Now, what is the most general of all, the most helpful of all, and hence for the school the first of all?

Nature and human beings surround the child continually. They constantly carry around it currents bearing every species of mental nutriment. Would you prepare for it another mental nutriment than this which offers itself to the child spontaneously? Even supposing that you were able by a strong incitement of the phantasy to alienate it from its own experience, would you desire to do so? Suppose the child would allow its mind to be filled with African animals, Roman emperors, mountains in the moon, and angels in heaven, would an intelligent, able denizen of this world and would a character conscious of itself be the result? You understand, of course, that in the last resort I desire to banish from instruction neither the African animals, nor the Roman emperors, nor the mountains in the moon, nor the angels in heaven; only they and all that is remote and as yet strange to the child is to be arranged into connection with what is near and of everyday occurrence, so as to cast a light upon the latter, expand it, freshen it, complement it; it is not to

thrust itself into the place of the latter so as to erect instead of the actual world of his occupations and duties a fantastical stage in the head of the child for idly hovering dreams. In this respect everything depends on the position of instruction; it should occupy such a position as ever to leave at the centre that which impresses itself on man most profoundly and most certainly; such a position as to have these deepest, surest, impressions also the truest, sharpest, and most exact, in order that the daily experiences of the child, the boy, the youth, and the man shall always find open gates and well-made roads to the head and the heart, so as to stir tongue and hand as the duty of the moment may require.

It is true, the external world, the daily environment, seek of themselves entrance to the mind of the child by the eye and by the ear. But they very often bar their ingress by their own multiplicity, variety, manifoldness. Human beings speak quickly. Into a single syllable they compress many sounds, into a few words many thoughts. Nature in a single meadow shows many forms, in a single flower many colours. Implements in the house are movable. They frequently change their position and their use. That this confusion is penetrated by the little child because it is urged on by a vivid need I admit. It makes itself acquainted with the things; it learns to understand language and to make itself understood by language; it learns to determine by the eye the place whither it must reach in order to seize an object. Now, we say, it knows how to speak. It never occurs to us even to say it can also see now; just as if everybody who is born with his eyes open knew by the fact itself how to use his eyes. Nor does it occur to us to teach children how to hear, though every day we experience the consequences of this neglect by being shown that far the smallest number of

people have a musical ear, i. e., know how to apperceive the differences of tone.

But, I ask, is the child actually able to see, actually able to speak, actually able to understand language, now that it knows how to express to some extent its animal needs and to guide and direct the hands by the eye, now that it has just freed itself from the first and intolerable pain of being incomprehensible to us and of being optically deceived itself? Are the avenues really opened by which Nature may stream in and human society communicate with the child? And in the course of time do our children as a matter of fact by and by spontaneously acquire the keen and exact eye of the savage and the elegant and felicitous expression of the Greek? When the child with slipshod manner caused by drowsiness barely glances over the surface of things so as to discriminate them, should need be, from each other, is not the wealth of forms that Nature surrounds us with wholly lost to him? Is it likely that in the future he will be disposed to learn how to use an implement with accuracy if he has never observed with attention its form, and how this shape suits with and fits into other shapes? Do you believe that your children will like to impress upon their minds definitely the details and magnitudes of the countries on the map; that your boys will take an interest in learning natural history, technology, mechanics, geometry, physics? Do you believe that any boy is well prepared for his trade who ceases practising and educating his eyes as soon as they may, if need be, help him out in the primary human needs?

Again, what stands so long and universally in the way of human education as lack of language? Who is more surely excluded from the benefits of instruction conferred in human conversation than he who neither

knows how to choose the appropriate expression nor how to appreciate the force of an expression well invented? Does even the educated man ever come to the end of the study of language, the creatress of all conversation, all society?

Exactly at the time when the child is still in motion, making language for itself, impressing forms on its mind, when need and exertion have not ceased in it, when it still asks for names, when it still finds every day new objects in its surroundings, inciting it to inspect them accurately, to look at them from all sides; now, before this natural progress comes to a standstill, now, when it is already slowing up, already inclining to immobile inertia, now is the time for succour; now must we perfect the opening up of the senses to shape and speech, in order that ability may be acquired to perceive nature and to understand the thoughts of men.

Above all the other senses it is the eye that must show things ere they can be named and discussed. Practise in sense perception, then, is the first of all, the most helpful of all, and the most general educational discipline which above we were looking for. Such exercises have been recommended by many. Pestalozzi, as far as I know, is the first to insist that this and no other instruction shall even at school, even in the lowest grade of the village school, actually occupy what is its due, the first and foremost place in all instruction.

His A B C of Sense-Perception, a collection of lines and figures which can be easily recognized, which can be reproduced definitely in drawing, which will recur to the child in almost all the objects of Nature and the implements in the household, and which, consequently, may serve for preliminary exercises in measurement by the eye, he places emphatically in the van of all his educational ideas.

Highly as I esteem him for so doing, yet I should like, at the hint of the science of forms, to withdraw very softly his equilateral quadrangle and substitute for it a sequence of triangles, which I should think would be a better auxiliary for carrying out his own idea. Of this we shall speak further, if you please, on a future occasion. You need be afraid of my triangles as little as of his quadrangles, even if two or three trigonometrical terms slip in.* Besides this A B C of Sense-Perception, and besides a line of suggestions for exercises in language which he has not made known with sufficient definiteness, you will find indicated in his book a third very general means of all learning, which together with the other two is to constitute the foundation of all other instruction. It is practise in the use of numbers—the art of reckoning. That the art of counting is of very general usefulness, that without numerical conceptions any considerable number of things would overwhelm us mentally, that no division of a whole however slightly complicated, would be apperceived by us distinctly, and so on, is plain to you without argument. And yet you, and with you the greater part of educated men, see but little of the vast realm of numbers, of the manifold, extremely artful combinations of the same, to which the study of Nature, even of common, every day nature which lies before the eyes of all, has driven investigators. This is the reason why I have said little of this elementary means of instruction, though quite essential. Pestalozzi, too, is very brief on this point. The exercises in mental arithmetic which have been long cus-

* Pestalozzi looks upon his A B C of Sense-Perception especially as preliminary exercises for drawing. In the posthumous works of Raphael Mengs, Halle, 1786, vol. iii, pp. 200 ff., there is a highly emphatic recommendation of such pre-exercises for drawing geometrical figures, for art-students.

tomary in many German burgher schools no doubt carry out this idea more completely than he intimates it, but it is hard to believe them there placed in correct relation to the whole of instruction as they are in the school of Pestalozzi.

To further the converse of man with his world is, then, Pestalozzi's primary purpose. That by this means the external activity of man—every species of gainful occupation—is facilitated, is obvious. But is anything done by this means for morality? That, perhaps, has been done tacitly which is presupposed by the moral lessons, the pathetic stories, and all the various excitations to ethical feeling, of which many—and a part of them excellent—have been composed for the benefit of children. The soil they are to occupy has been prepared. That is to say, the man or the child whose eye and ear are given up to Nature and human society is to that extent withdrawn from the sensations within himself, in other words, from his own pleasure or displeasure; egotism is rendered impossible in one that pays attention not to himself but to the relations of things and of other persons. Such a one is prepared soon to regard himself as only one of these people collectively, as one among many, and thus he will soon find his appropriate place. The habit of taking a general view of the relations of many, when it has become the chief tendency of the mind, of itself carries with it a love, never more to be lost, for order in these relations and for the maintenance of such order by justice and morality. Afterward, when these roots of the moral sentiment have grown and spread, comes the time for the direction of man's attention to himself again, in order that he may try to gain power over himself, analyze, and purify by watchful criticism his sentiments, and enlist his powers wholly in the service of the general purposes which he has recognised.

You will not carry what has here been said to the extent of supposing that in the school exercises which have been indicated there is a miraculous force for rectifying with certainty the character of every child. In respect to morality, perhaps more than in any other respect, every human being, independently of education, inclines in his own quite peculiar direction. Consequently, there is a corresponding need for an especial nurture and care for each person. To this, attention may be called, it is true, by general rules, and preparations may be made by general means. But the exact determination of what is to be done in specific cases must be committed to the delicate and profoundly reflective judgment of a near observer. It is true that a general view of the relations of many people is always the true basis for morality; but in order to give to childhood this view, the Pestalozzian exercises will be insufficient with one child, while with another they will hardly be necessary. None the less, by them the direction is indicated whither the educator should turn his first more definite endeavours in the formation of character. But—how far we have passed beyond the sphere of the Pestalozzian reform!

Let us return to it. What has thus far been developed relates only to the very first beginnings of that series which we seek. As to the continuation and completion of the same, Pestalozzi has as yet left us very much in the dark. However, you will find, now and then, in the book some remarkable points. To expatiate on them would not be to the purpose in this essay, which is intended only as an introduction to the book.

Another matter remains to be considered. We must determine by a comparison how the totality of Pestalozzi's branches of instruction is related to the whole scope of your own educational equipment. No doubt the former

is incomparably greater in respect to the number of people whom it is intended to serve, but the latter, on the other hand, is incomparably greater in respect to the number of activities, of considerations, and of reflections which must be provided for in it. In what has preceded we endeavoured with Pestalozzi to meet the most necessary wants; a mother desires to do more for her children. The most necessary things vanish—often only too much so—among the many things which she would like to accomplish, and actually, under the favouring conditions of the higher ranks of society, can accomplish. If these many things, if the whole of an education equipped with every auxiliary, had been the subject of discussion, we, seeking for something else, should have found something else, and should have found it by an entirely different line of reflections.

In the preceding we found the most general as being that which facilitates the most of what is necessary. Starting from this point, we found exercises in sense-perception, speaking, and counting as being the most general preparations for the apperception and use of that means of education which every one possesses, which thrusts itself upon every one, no matter of what rank and position—to wit, his daily experience. Thus we saw the generality of mankind in a state of need. In such a state the lower classes of the population actually are. By way of alleviation we must bring in all haste simply the largest supplies we can lay our hands upon, the most solid, the most substantial. To the extent that the actual needs are common to all human beings, it was for us also a preliminary care to provide against possible sufferings from a deficiency of the necessaries. To the extent, however, that a higher culture is for us the larger and more difficult problem, you have, I am certain, observed ere this that we require a far

greater delicacy of feeling with a far larger horizon, a far richer imagination. We, in addition, require an apt eye for far greater profundities of investigation. All this one has not hitherto dared so much as offer to the generality of mankind for fear of endangering its work. These results can only be gained through education by a procedure which, to be sure, comprises the preceding, but which nevertheless starts from a main point of view that is different. You ask, Which is this main point of view? Which is this first object giving to the educator the thoroughgoing rules for his whole enterprise, and more especially for determining the series. It is easy for me to write down a word. Let him care at all points for the possibility of ÆSTHETIC PERCEPTION. But for you this word is hardly plain enough; and some men will set against it exclamation points. Nor do I write it down to any end, except to indicate somewhat more distinctly by means of contrast the one-sidedness which Pestalozzi in the pursuit of his purpose was neither willing nor able to avoid.

CHAPTER III.

HERBART'S pedagogical activity in the leading families of the city republic of Bremen did not, of course, remain without influence on the city schools. To enter into details lies, however, entirely beyond the scope of this work. Nor does it come within its compass even to enumerate the titles of Herbart's lectures, delivered before the Bremen Literary Society and at the Bremen Museum.

One of the latter addresses suffices to state his final judgment on the sphere and value of Pestalozzianism. The address was delivered after Herbart had gone to Göttingen and while passing through Bremen on a journey. It was

published in pamphlet form. The reference to his previous addresses to Bremen audiences in the opening sentences is obvious.

That Herbart's mind is no longer in a tentative state as to Pestalozzianism is equally evident. This ripening in Herbart's opinion is due partly to Pestalozzi's publishing in the preceding year, 1803, three important works on his system; namely, the Mothers' Manual, the A B C of Sense-Perception, and the Objective Teaching of Number Relations. In larger part, however, as the reader will feel, though he may not be able to analyze the feeling, it is due to reflection on the subject. Herbart's really profound parallel between the obscurity of Pestalozzi and that of Fichte contributes to produce this feeling or conviction in the reader. It shows that his psychology as well as his pedagogy had been brought to bear on the problem of Pestalozzianism. In the preceding article, as the reader will remember, he had not gone so deeply. He dwelt more on the surface. He glossed over the Pestalozzian obscurity by attributing it to the haste of an old and busy man. Herbart's rejection of pedagogic exclusiveness also shows that he had taken a larger range of vision than most of his contemporaries, who insisted either on championing Basedow's philanthropinist method, or on joining Pestalozzi in his search for the true order of studies.

If further evidence were needed of the rapid evolution of Herbart's thought, it is furnished by his pronounced opposition to the doctrine of transcendental freedom preached by Fichte, his great teacher. As we have said, this evolution, so important to the history of pedagogy, is mainly psychological. Herbart recognised that the German, the Englishman, the Turk, the Negro, and the Samoyed, are not, psychologically, the same man. He is ready to begin his greatest negative task in psychology, the demolition of those artificial pigeonholes, the so-called faculties, into which the human mind was then supposed to be divisible.

For public educators in America, whose attention is vividly drawn to moral education by criticism on the alleged insufficiency of the public schools in this respect, interest in

this point of Herbartianism centres, of course, on Herbart's showing the connection between re-presentations (or ideas) and character-building. Two traits in the treatment we would especially emphasize: First, the plain inference that of this as of other problems, EDUCATIVE INSTRUCTION is the solution; second, the reminder that success in moral, full as much as in intellectual, education depends on the proper psychologic grading of the training conferred. This is a vital condition under any system of pedagogy. And yet we find it daily and hourly forgotten in the schoolroom and elsewhere by zealots in ethical training.

On the Proper Point of View for Judging the Pestalozzian Method of Instruction.

A Discourse delivered by invitation at the Bremen Museum.

A transit like my present one through Bremen does not, it must be admitted, afford time for preparing a regular public address. Yet I have been given a friendly call, to which I have the more pleasure in responding, for the reason that several years ago I had an opportunity of personally experiencing the indulgence of the honourable public which is here assembled.

The request made upon me has not been simply for a discourse, but a subject has also been assigned to me. I am to speak of the Pestalozzian method of instruction. By this time so much has been spoken and heard on this subject, the public by empty trumpeting has been put into such a tension of high expectation, while on the other hand dry elementary books have disheartened it, and I myself have already upon so many occasions spoken my mind by tongue and by pen, that for me at least the time has come, perhaps, for silence. Since, however, I am reduced to entertaining you with a rhapsody of thoughts, we may perhaps as well start from that which has been often discussed,

whence an unconstrained association of ideas may haply drift us on to interesting points, since assuredly a number lie in the neighbourhood.

It is not good for the Pestalozzian cause itself to keep any one gazing at it alone. In the inventor's mind it hangs together with all sorts of concepts and endeavours, never distinctly given utterance to by him. Men deferring to advanced age the beginnings of their activity, usually meet with the fate of being unable to find their way out of the multiplicity of their accumulated ideas and intentions. One has just passed away from among us—Kant. He would no doubt have avoided the burdensome prolixity of his writings if he had come forward at a time when his investigations still remained more of a novelty to him, and he with greater ease thought each one separately, and before, in connection with them, he had invented so much terminology. The need of labelling everything by names and technical terms does not arise until the multiplicity before the mind becomes too large for every detail to be known and discriminated distinctly by its specific shape.

In the case of Pestalozzi we have to add: he was too deficient in scientific auxiliaries, and perhaps still more in the cold-bloodedness needed in handling scientific tools, in properly heating and mixing the learned drugs, and in writing orderly recipes for our imitation of his art. True, he had at last to make the concession of presenting at least a few parts of his method in definite scholastic formulæ in order to have any hope of spreading that method. But he has done it with such stiffness as to make one think that he, the once popular author of Leonard and Gertrude, praised for his beautiful, vivid, attractive style, is metamorphosed into a pedantic drillmaster in arithmetic, pleased with himself for having

filled a thick book with the multiplication table. A comparison with a very famous philosopher again obtrudes itself. Fichte, by a few anonymous writings, made a host of fiery adherents and equally fiery opponents. Both agreed only in finding the force and clearness of his language worthy of amazement. Yet the same Fichte took on an exasperating resemblance to the obscurest of the Schoolmen the moment he became the author of the Science of Knowledge. Strange, indeed, that men most full of life assume the driest tone when they have it at heart to expound their very depths of thought. Goethe has said : He who has thought that which is profoundest loves that which is fullest of life. You see that the obverse dictum holds good also : Intensity of life has a peculiar inclination and ability for plunging into the profoundest depths of thought. Admitting that what comes from life leads to life, the dry methods, it is to be presumed, are only a dark passage out of light into light.

If we look upon it that way, if our youths and boys be submitted to an exertion similar to that practised by such a man in taming his own powers, the constraint which he imposed upon his imagination might be of equally beneficial effect in their case. For, after all, the whole of education does rest upon the early habituation of the supple boy, the tender child, to those mental and physical movements which, out of all the trials and endeavours of man during many centuries, we have selected as the best and the most profitable. The best and the most profitable—"Ay, there's the rub!" That is the very point in the dispute. Are the Pestalozzian forms better and do they profit more for the purposes of instruction than the amiable, varied entertainments upon which we were recently congratulating ourselves at our having been so fortunate as to introduce into our schools? That is exactly

what people call in question—whether to keep the children for months on a description of the human body while they carry about with them constantly the reality to which the description relates, be better than to give them useful and agreeable geographical knowledge by which they acquire concepts of the world, its magnitude, beauty, and so forth. At this point I must, gentlemen, above all, enter a lively protest against that way of stating the problem. This "whether" and this "or" is not my way of looking at the question by any means. The main proposition I shall stand upon is, that neither method is to exclude the other. However, not to become too serious, I shall, being a traveller, with your permission take a brief journey into another territory, a territory upon which the art of education has the fortune, or, if you like, misfortune, of closely bordering—I mean the domain of philosophy.

In the first place, let us cast a glance at the variety found together in the mind of an adult person. It consists of knowledge and of imaginings; of decisions and of doubts; of good and bad, stronger and weaker, conscious and unconscious sentiments. Its composition in educated differs from that in uneducated man. It differs as between Germans, Frenchmen, Englishmen, Turks, Negroes, and Samoyeds. The manner of its composition will determine the individuality of the man. Education desires to act on it, building and improving, only it does not exactly know how to take hold, and to what extent it dare have confidence in itself. Let me ask, does the principle of a man's education lie in himself in the sense in which the whole shape of a plant lies prepared in its germ, or does the construction of his individuality originate in the course of his life only? The latter assumption would be somewhat like assuming that, under appropriate condi-

tions, a lichen during its growth might become a moss, a moss by degrees a grass, the grass a shrub, and the shrub a fruit tree, or reversely. This assumption would imply that none of these organisms is in itself a finality, but that it is open to changes in its internal structure by external accidents, exactly as the act of the gardener does, in fact, change many flowers so that they become double, although they were by nature single, and so forth. On such an assumption the art of the gardener would be a far greater art than it now is. We should demand at its hands regularity and beauty in the shapes of plants. Gardeners continuing human, after all, they would also, no doubt, incur many a guilt of negligence or even of ruination, which would not be readily forgiven them, since a thing so perceptible as a plant immediately betrays to all eyes every malformation which it has assumed. Furthermore, according to the different taste of each nation, we should find flowers and trees different in Germany, in France, in England. Every nation would prescribe the cut of its trees as carefully as fathers to-day endeavour to educate their sons, and good patriots the rising generation of their countrymen, after their idea or even in their own likeness. I hope I have made myself understood.

Let us return to the question. Does a human being bring with him into the world his future shape or does he not? In respect to his body he doubtless does; but that is not our question. We speak of the mind, the character, the interest, the entire disposition. Here we meet, as you are aware, a host of opinions. A man's temperament is bestowed by nature, say some. Naturally, man is good, say others. But, by original sin, "born evil" is added by a third group. It is education that makes everything of him is the opinion of a fourth judge. He makes, posits, and determines him-

self, exclaim the latest systems, forgetting as they do so that they themselves in other places have declared all sensuous existence—both of the inner and outer senses—to be a pure product of natural necessity, and that they were bound so to declare. The last-named party is disposed of the most easily. Such philosophers have closed their intelligible world against every influence; it is to be wished that they would also avoid letting any influence emanate from it, in order that our sensuous world—i. e., everything that we can in any way discover in our consciousness—may go quite undisturbed on its own way. We should then impute to the sensuous man what was done by the sensuous man, and what he ought to do would be made the the object of education and of the social institutions, which, when all is said and done, are in the power of man, whereas nobody can do anything with the intelligible world. Let us leave aside, then, this mere dream, which psychology is bound to declare a delusion, ethics a misunderstanding, and metaphysics an absolute impossibility.*

Let us turn to experience, for there is not now time for a lengthy disquisition. We find in animals instincts. We find even in the lower animals art impulses; whence the similarity in the lives of all bees, and in the lives of all caterpillars of the same species. These animals, it is true, have freedom of motion, but the interior irritation or promptings everywhere accompanying them give them no respite. They must fulfil the work of their nature. They have impulses only because they are impelled by irritation. They continually act on the same impulse. Their action is to the purpose, it is consistent merely because the irritation continues always the same, or, at all events, changes in

* My remarks are strictly applicable only to transcendental freedom, not to the intelligible world in general, nor to freedom in general.

them periodically only according to a natural rule. Much more consistent, even, is the internal action of a plant. But much more inconsistent is the action of man. He is rational in the place of instinctive. He is impelled by the mechanism produced by the re-presentations or ideas that he has apperceived. These re-presentations themselves are forces which check and aid each other. They constitute powers which elevate and throw down, oppress and liberate one another, and which by this very conflict get into all those conditions which we call by a name far too general, far too indefinite—will. How much there is implied in this expression will! Inclination, desire, fear, courage, choice, whim, decision, reflection; good will not knowing what is good, ill will imagining itself good; at another time, insight without resolution, a resolve without force; the detestation of a crime which in the same moment is knowingly committed; and all the other phenomena in which the marvellous mixture and isolation of the first named, their incessant and continuous mutation and new formation, every instant put to shame and annihilate the divisions made by the philosophers between the understanding and the will, between the reason and arbitrary choice, between the impulses and human freedom. The significance of these terms it never has been possible exactly to explain, to determine, to limit. They are approximate designations of vacillating positions in a machine which in the course of time is incessantly building itself differently, and therefore incessantly changing in action and tendency. Only do not let us forget for a moment that this machine is exclusively built of re-presentations or ideas. Hence it has no tendency but a tendency into actual re-presentation, does not attain anything by all its internal and external activity and efficacy but new representation, and suffers nothing from without but the

checking of old representations. What I am saying this moment of man every man would every moment say of himself, were it not for the one troublesome little circumstance that representation does not represent itself but its objects, just as the eye sees not itself but the things around it. Could the eye see its own seeing, man would be able to perceive immediately that he wills only representations and knows only representations, or, to speak more exactly, that his knowledge is only a perfected and his volition a checked resurgent representation. Such seeing and perceiving will happen when the Weser River flows upward, the lions and lambs play together, and the latitude and longitude of Liliput and Brobdignag are determined with precision.

I daresay we have philosophized long enough, at least for an occasion where we have no opportunity to offer proofs, but only to proffer assertions. Let us return to education. If the truth of my assertions be granted, education will endeavour to nourish man by representations. It would determine upon composing him of them altogether, had Nature not decided for the most part what representations a man is to have, and if Nature, after all is said and done, did not have to furnish the objects, as it had, in the first place, to furnish the representing being himself. Nature, however, is kind. She both gives to man and yields to man. It is this that provides the work for a Pestalozzi and a Basedow. As many plants and animals are nourished by the sea and earth, so many pictures are heaped about the child. All the mischief and folly ever perpetrated by arrogance and delusion, invented and noted down by industrious scribes, and copied by monks doing penance, furnish so many tales ready at hand to satisfy the curiosity of children and incite the impetuosity of boys. The means of education

displayed for our free disposal are plentiful. It is their very multiplicity that embarrasses us. The impressions from the natural world and the world of culture that carry their currents around the child are in such number as seems almost to make the art consist more in keeping them off than in bringing them on. It is in this regard that if accident be allowed full sway it makes of every individual a separate being. Nay, it alternately builds up and destroys in the same child. It sets the individual at variance with himself. It sets men at variance with one another. It could not do so were human nature firmly laid out in its plan, like a plant or like every animal body. Such a predisposition circumstances, to be sure, might favour or check, but they could not mark it with contradictions such as we find in man and society. But for the very reason that human power only elaborates what it receives, it is of such great importance what we give to it. Therefore, education must regard its office in very essence as consisting of giving and withdrawing. Education by no means consists merely in supervising and tending, like our gardening art that makes only plants its care. In the case of the latter the one essential point is to bring about favourable and keep off unfavourable circumstances, and to have rain and warmth, the soil and the atmosphere well suited to each kind of plant. Man, on the contrary, requiring no determinate climate but making his way in any one, capable of becoming, as you will, a wild animal or personified reason, and formed incessantly by circumstances, needs an art which shall build him up and construct him in order that he may receive the form that is right. And the form which is right is that which in the future, when he shall comprehend himself, will please him; which, when others contemplate him, gains him their assent; and which, when with them he is to con-

stitute a social whole, enables him to join them with accuracy and effect.

Let us assume that art or accident, no matter which, has begun, and is continuing to do what Nature does not do. Let us suppose a person in a condition of half-education, and with one-half of his plasticity still open to influence. In this intermediate state the human being has obviously approached the plant. Already something exists which, if unhindered, develops in a definite way, and which in an equally definite way helps or resists any new accession. Obversely, too, any new accession must now so conform to what already exists as to be helpfully promotive of its further thrift—granting, let us suppose, the desirability of thriving. The art of continuing an education already begun grows, therefore, more and more similar to the art of gardening. The blessings of education change more and more into mere almsgiving. The treatment changes more and more into a touch as of a breath. True giving and withdrawing diminish. In a child's mind a definite interest may be implanted. The interest of a youth can only be fostered. A child believes what it is told, thinks what it has heard, does what it has seen; we build a world for it by pictures and tales. In a youth, on the contrary, we can only widen or narrow the world in which he lives; in it he builds himself a hut, disdaining a palace built elsewhere against his sentiments.

If these are known truths, then I should like to ask why the spirit of the Pestalozzian method is deemed a riddle, and why there is still doubt as to its worth and the place appropriate for it. I should hope no one is so far in error as to think it mainly hinges on the well-known descriptions of the human body, the horizontal lines, and the paraphrase of the multiplication table. As regards the subject-matter of instruction, pedantic limitations

must not be thought of. The whole field of actual and possible sense-perception is open to the Pestalozzian method; its movements in it will grow constantly freer and larger. Its peculiar merit consists in having laid hold more boldly and more zealously than any former method of the duty of building up the child's mind, of constructing in it a definite experience in the light of clear sense-perception; not acting as if the child had already an experience, but taking care that he gets one; by not chatting with him as though in him, as in the adult, there already were a need for communicating and elaborating his acquisitions; but, in the very first place, giving him that which later on can be, and is to be, discussed. The Pestalozzian method, therefore, is by no means qualified to crowd out any other method, but to prepare the way for it. It takes care of the earliest age that is at all capable of receiving instruction. It treats it with the seriousness and simplicity which are appropriate where the very first raw materials are to be procured. But we can be no more content with it than we can regard the human mind as a dead tablet on which the letters remain as originally written down. The entertaining or conversational method which in the main dates from Basedow has the peculiarity, in its way an excellent one, of seeking to adapt itself to the natural movements of the child's mind. It should therefore follow close upon the Pestalozzian method when the latter has finished. In practice both methods require to be fitted to each other. Here is the gap which so far remains unfilled. It will be filled, however, if we are patient. Bremen is so happy as to possess men of whom it may be hoped that by a pleasant and rare co-operation they will give the first example of such a many-sidedness as is required by the extremely different periods in the age of the human being.

CHAPTER IV.

AFTER resigning from Göttingen to accept the distinguished chair at Königsberg, occupied before him by Kant, and after him by Rosenkranz, Herbart, in 1810, founded a pedagogical seminary. In addition, he was a co-founder and for a while the leader of the Pedagogical Society.

As we are about to present a specimen of work done at the latter, it seems wise first to understand the spirit in which its discussions were conducted. To this end we ask the perusal of the following paper, which was found among Herbart's manuscripts and posthumously published.

ON PEDAGOGICAL DISCUSSIONS AND THE CONDITIONS UNDER WHICH THEY MAY BE USEFUL.

I.

It is the rule for everybody who is asked for an opinion to have one; for if he had not, he would invent one. As many people as find opportunity to talk on a subject not exactly involving Greek or Arabic or the integral calculus, are incited to set up some view, no matter how one-sided it may be, as their view, and to adorn it with words.

The endeavour of every one in a disputation to assert his proposition rarely results in convincing others, but usually in the confirmation of his own mode of thinking. Plausible arguments can be found for everything; as they grow in number, so grows one's partiality for one's own invention.

These evils increase the more a subject is of the kind for which decisive conclusions are difficult and yet for which the appearance of reasonableness on all sides very plenteous. Pedagogy stands in this predicament. Everybody

has seen and experienced something of education, at least in his own case. Every one has drawn from social intercourse, from history, from would-be philosophical reflections and aphorisms, now supplied by every elegant periodical, some sort of opinions concerning the destiny and educability of man. Such opinions are intimately connected with his feelings, with his way of thinking and acting. In his pedagogical opinions he represents himself; in defending them he defends his own personality. Whatever has been said and written concerning pedagogy is judged by every one according to his feelings; but the unsafe character of emotional judgments is well-known.

What will become of the case when pedagogy gets to be the talk of the day, and many are invited to participate in the discussion? 1. A multitude raise their voices simultaneously, every one full of self-confidence and but little inclined to hear. 2. Others listen, but have soon made up their mind, and have only been deprived of appetites for thorough investigation. 3. In practice, the ability of each one decides. Each follows his own opinion, as far as circumstances permit. The result is mingled with a number of adventitious circumstances; they falsify the teachings supposed to be derived from experience. Still more falsifying is the admixture of one-sided apperceptions of experience and of surreptitious conclusions. 4. Finally the whole subject appears as a matter for decision by majority vote.

This natural course of things in respect to pedagogy is elucidated by the fate of philosophy, if not of the sciences generally. The more the public has chattered about philosophy, the deeper has this branch fallen. Even that there are so many less studious people now than formerly, has, I doubt not, its main cause in a certain widespread shallowness, the result of sham culture. Granting that

pedagogy is a science, such as must be spread to enable it to be useful, it is, nevertheless, endangered by a precursor and substitute consisting of a crowd of contending opinions. For many people it would be better never to have heard anything about pedagogy until they could be reached by sound instruction, and many errors are avoided by not insisting on people's saying something when they know nothing profund to say. In a democracy and during revolutions the political opinion of many is asked, and arbitrariness and fancyings accordingly arise instead of sound reflection; but in the state something may always be left for arbitrariness to deal with. In science, and consequently in pedagogy, there is simply no room for it.

II.

Under what conditions can pedagogical discussions, nevertheless, be of use?

1. Principles must be assumed from which arguments can be developed, and by which they can be tested.

(*a*) Principles concerning the ultimate end of education and instruction as well as the purpose for which schools are founded. Such principles depend on the profounder principles concerning the worth of man and the business of the citizen. He, for example, who believes that schools are founded to stamp officials of various grades can never agree with him who insists that educated men should impress their stamp upon officialdom.

(*b*) Principles as to the educability of man. The disputation, for example, of one who believes that a northern climate will not bear Greek and Roman culture—or, conversely, of one who believes that what the heroes and sages of Greece lacked in the way of pedagogic auxiliaries is of no use—will not instruct one who holds that the educability of man is founded in the universal and principal

traits of human nature, external circumstances being but a coefficient in which the lack of one thing is often compensated by something else, our business being to arrange everything in the most advantageous way possible under the circumstances.

2. No one must demand a vote who has not pedagogical experience.

(*a*) This experience must have been acquired with children of different ages, up into the later years of youth, and with individuals of different dispositions and education; for no age displays the quality of another.

(*b*) The experience must have been gained from single, long, and sufficiently observed individuals. Otherwise, it is impossible to see the inner nature. In schools where the teacher can enter little upon the training of individuals, all appear much less educable than, in fact, they are; for only that small amount of educability is revealed which obeys the short and rapid touch which the teacher can expend upon any individual. In order to observe the reciprocal, very strong influence of fellow-pupils upon one another, the teacher must be a thoroughly practiced observer or it will escape him entirely. Speaking generally, a school-teacher is always inclined to look upon his class as a historian does upon a nation, namely, as a crowd of people which one must seek to gain a total impression of. This total impression falsifies the apperception of every individual.

(*c*) The interpretation of one's own experiences must not have become a habit of representing to one's self all boys and youth to be like those whom one has seen; all results of methods to be like the results one has obtained from one's own method, but the single actual specimen must have been contemplated and reflected upon as surrounded by other possibilities; during experience one

must have noticed all that must have turned out otherwise, if this or that circumstance had changed. But for those precautions, different people will always have different experiences, according as they set about it, and each one's vigorous assertion of his own experience will not in the least refute another who has an experience too.

3. The disputants must not be more than can enter into sufficient mutual explanation. When the number is so large that either one must take the whole time or each one may speak only a few words in order that others also may have the floor, misunderstandings originate from insufficient utterances and annoyance of false interpretations which one does not get the time to rectify. Hence the number of conferees should grow but gradually. The first who come together must have come to an agreement among themselves, if the accession of others is to be of use.

4. Rulership should not be accorded to the maxim of polite society that no subject should be exhausted, but the seriousness of thorough reflection must be adequate to the importance of the matter.

CHAPTER V.

In June, 1814, Court Preacher Zippel read before the Pedagogical Society an essay, the essential contents of which may be surmised from the tenor of Herbart's Observations upon them, which we shall next present. They show us Herbart actually at work in pedagogical discussion. Besides, much of the Observations remains as applicable to-day as when first uttered.

The cry against the multiplicity of studies, as the reader will see, is an old one. At first sight Herbart's insistence on many-sidedness seems to make matters worse. But it is,

indeed, the very reverse of a sanction of the existing confusion. In the first place, Herbart, who more nobly and more carefully than any other man has claimed for the school all that it can really do, sets forth with equal carefulness its limits. This limitation removes at a stroke the pernicious tendency to regard the school as a factotum.

It is worth while pointing out that this very tendency stands sponsor with us for two exactly opposite evils. We are prone to an overlaudation, which is an invisible barrier to a true appreciation of educational needs. But we are equally prone to indulge in a vastly diversified multiplicity of complaints, which display with equal clearness the lack of an exact estimate of the limitations to the efficiency of our teachers and schools.

Till critics perceive the line of delimitation pointed out by Herbart, they will inveigh against the schools for conditions for which the schools are in no way responsible.

In the second place, Herbart proclaims the doctrine that the pupil has as good a right to his free time as the teacher has to his or hers, and, consequently, he insists on the utmost possible economy of time; subjectively by intense intrinsic interest; objectively by the organic connection of instruction. To-day courses, teachers, and students are still more overburdened, and the revolt against mechanical teaching and examination crams is gaining ground everywhere. Hence, every word he says on the subject is worth thinking over. We are listening to a scientist who confesses to twenty years' toil before gaining an insight into even the foundations of a real psychology.

More than this, we are listening to a thinker to whom the negative results of metaphysical endeavour and the very positive results of mathematical investigations had taught the lesson which no practical teacher can bear in mind too persistently—namely, that neither a fanatical insistence on unity nor an arbitrary separation of correlated and cognate subjects can end in any way but disastrously, while the prize of success is for him whose painstaking investigation has taught him just how and what to articulate in his in-

struction. In other words, we are listening to a man who was highly successful in the direction of education, and thoroughly alive, therefore, to pedagogic experience as a check on pedagogic theory; a man, consequently, entirely unlikely to mislead one in favour of pedagogical abstractions.

This eminent soundness of pedagogic insight is, perhaps, most conspicuous in his insistence on equilibrating many-sided interest; the sole position tenable by the well-balanced pedagogist against numerously besetting faddists, and the position which, in duty to the best among his students, he is bound to take.

The concluding paragraph will be of especial interest to teachers who are vacillating on the question of accepting or rejecting school-readers composed of excerpts.

By far the most important section, however, is on religious teaching. The reader will perhaps be astonished to see how, like pedagogy, political history repeats itself. The position of the Rev. Mr. Zippel is, the difference of time and country apart, the position of those who attack our public schools for neglecting religion. Considering the interval of space and time, it is still more surprising that Herbart's attitude is—and not only in spirit, but in the very amount and grade of religious instruction he suggests—almost the exact equivalent in substance, though not, of course, in form and details, of the condition which exists to-day in American education, taking that term in the broadest sense so as to include both public and Sunday schools.

The argument, as usually with Herbart, is put very compactly. Whichever way the reader may incline, he will probably decide that the few lines containing the simile of the jewelry are themselves gems in pedagogic literature.

Observations on a Pedagogical Essay.

Read before the Pedagogical Society.

A very brief report of the contents of the essay before me might read as follows: The Rev. Mr. Zippel insists upon the simplification of instruction. In order to attain

it he would admit into our schools only one ancient language, no matter which. Much more time than heretofore is to be devoted to it and to religious instruction. Most of the other studies are to be carried on by reading, under directions from the teacher. Upon this dry announcement it might be said that the central idea is perfectly well known. Great, indeed, is the number of those who have held forth with zeal against the excessive number of studies. The means proposed for carrying into effect the demanded simplification might be deemed paradoxical and hardly applicable. But it is not so much the proposals as the sentiments which have caused me to value this essay. For this reason my observations will often refer to some striking turns of expression employed by the Rev. Mr. Zippel, and I shall have to recur to them in order to indicate the points upon which my glosses are made.

All-sided or, rather, many-sided knowledge—for a totality is constituted not even by human knowledge, much less by that part which the schools communicate—is, indeed, the purpose of instruction at school. That is to say, it is the nearest, the immediate purpose. The school can not be made an institution for the whole of education, least of all for the whole of the education of an individual. Of the sum of the forces capable of educating a man, it controls only a determinate part, a separate class—to wit, the sciences. What the world, example, converse, family, and, above all, the silent self-efficacy of mind working within itself contribute, is not under the control of the school.

It does all that is possible if it puts properly in motion the educative force which resides in the sciences. Besides, the receptiveness of individuals differs as to its direction. The school, on the other hand, must prepare

the opportunity for education for many. Therefore its instruction must not be interpreted as though all of it were intended for all in an equal degree.

Partly, it is true, the value of a good scholar does consist in the many-sidedness of the education he appropriates. But the excellence of a school does not show solely in the many-sidedness common to all its pupils. It shows quite as much in the diversity of specific excellences reciprocally distinguishing above each other the scholars that proceeded from it. The school of Socrates educates Plato and Xenophon, Aristippus and Antisthenes.

What is the value of many-sided knowledge? Is it capable of statement? This value is highly variable. It is determined by what each one makes of his knowledge.

That the soul learn to love the object of knowledge, understanding its value, relations, and connection; that readiness and art enter into the knowledge, or better, spring of necessity from it—is surely the main point. From this point, I admit, school instruction, by the very multiplicity it is compelled to comprise, is very apt, if interest becomes rooted at any time, to tear it away. I concede that frequently there is danger of robbing the scholar of the leisure required for self-elaboration. I not only concede, but it has been always the essential content of my pedagogical teaching, that this distraction is to be in every possible way prevented. But from this conviction I have never been able to draw the conclusion—as I look at the matter, a hasty one—that the quantity of subject-matter should or even might be diminished considerably. I reason in this way: Every method which needlessly uses up time that should have remained at the disposal of the pupil for free and individual occupation must for that reason alone be considered faulty. Economy of time is pre-eminently attainable by the greatest possible inten-

sity of interest being excited, since what we pleasurably learn we learn very quickly and apprehend profoundly. And, finally, the interconnections of human knowledge must be investigated in the most accurate manner, so as to enable teachers to set any interest once excited to work immediately in all directions, in order to accumulate the usury of learning on this interest as well as on the capital that has been acquired, and in order to avoid as far as possible such intellectual disturbances as might diminish the capital.

Solicitude lest human beings be sold like slaves into the service of the sciences is often, I dare say, felt by one who reflects on the effect of the severity with which what he has learned is required at the hands of the scholar when, instead of arousing his interest, the work is done for a gaudy examination. I honour this solicitude. I find it persists even with the best; nay, it persists even with an oversimplified course of study. The reason is that, in this regard, everything depends upon the teachers. If they are mechanical workmen, their pressure on the mind of youth will infallibly be the severer the greater their desire to demonstrate their faithfulness to their duties as teachers. The teacher must be a man with ideas if he is to give free motion to the ideas of his pupils. I recall as a single instance in point history. Nothing so oppressive as to have counted out a certain number of facts to commit to memory; nothing so stimulating to the youthful imagination as a story from history well told. In my didactic institute I have experiences of the sort before my eyes every week.

The question whether the means for educating learned specialists be also the right means psychologically is a very important but also a very seductive question. It is forgotten by every soulless educator, and answered pre-

cipitately by every arrogant educational reformer after his own individuality. Each deems every youthful nature to be exactly what he deems himself to be, having like spiritual needs, like limitations, etc. Every one without ceremony judges others after himself. For this reason so many imagine themselves able to teach pedagogy, and to need it no longer for instructing themselves. As for me, I have for twenty years employed metaphysics and mathematics, and side by side with them self-observation, experience, and experiments, merely to find the foundations of true psychological insight. And the motive for these not exactly toilless investigations has been and is, in the main, my conviction that a large part of the enormous gaps in our pedagogical knowledge results from lack of psychology, and that we must first have this science—nay, that we must first of all remove the mirage which to-day goes by the name of psychology—before we shall be able to determine with some certainty concerning even a single instruction period what in it was done aright and what amiss. The mistakes that can be committed in a single recitation, if taken by themselves, are, to be sure, trifles. But they accumulate enormously if repeated at every new recitation.

These thirty years almost, unity in that which is manifold, the highest unity of all subordinate unities, has been the watchword of all who have ever occupied themselves with philosophy. Reinhold looked for the highest unity in a single principle of philosophical knowledge. He lauded with enthusiasm his principle as the one thing needful. The requirement people had faith in, but the principle they disdained. Fichte found a far more forceful unity—the Ego. Now, it was opined, the great work of Kant was on the verge of being carried to completion, as the Kantian philosophy—and what a variety of things

besides!—might be concentrated in the Ego. But physics could not be got in conveniently. Schelling accordingly invented his Nature philosophy, and in connection with it the Absolute. And then public confidence in philosophy fell by rapid degrees deeper and deeper as the artificialities for pressing everything into the new unity grew odder and odder. True philosophic thinking has on this occasion gone out of fashion altogether. Most of those who talk of philosophy have forgotten their logic, which amounts to speaking of the genius of a language without knowing its declensions and conjugations any too well. And thus it is that we have to-day celebrated authors who will have no more of philosophy, but instead religion and mathematics and art. But, unfortunately, all these things, very valuable in themselves, are not philosophy. Will the striving for a highest unity, which has been the ruin of philosophy, work any more favourably in pedagogy? Will fewer artificialities be employed in the latter than in the former for the sake of pressing the manifold, which by its nature must remain mutually external, into one another? On the contrary, we shall see that the endeavour to set all at a single apex is bound to become as harmful to the educator as, on the other hand, the shredding and breaking up into piecemeal of that which really belongs together.

Both philosophy and education require throughout that every connection inherent in the nature of the subjects be recognised, and that it be qualitatively and quantitatively evalued neither at less nor at more than its real amount. What the amount is must be separately investigated for every specific kind of connection, for each special line of teaching, and on every fresh occasion. It is impossible to predetermine universally either the amount or the specific details at a given point. I can not help insti-

tuting in this connection a comparison with mathematics, which has been elaborated with felicitous success above all the other sciences. Mathematics strives for the greatest generality in its propositions, as philosophy and pedagogy do for the greatest possible unity in the manifold. But when has it been necessary to call out to the mathematicians of a whole age or even of a whole nation in respect to the entire science: Be careful not to exaggerate generalization! True, a few mistakes of the sort have happened, but they are receded from at once on accurate trial whether the proofs for a proposition are sufficient to uphold it with certainty for every case contained under a specific formula. The mathematician is aware that if too little generality constitutes a lack in our knowledge, exaggerated generality brings out something far worse, namely, false theorems.

Now, the Rev. Mr. Zippel demands that even a youth shall seize upon some subject with especial love. By substituting with especial *interest* I shall appear as if only putting a feebler word into the place of the stronger and more emphatic. But I confess that it seems to me that the right measure of expression resides in this instance in the weaker word. The child should embrace with the infinite plenitude of love his parents, and the youth his country. But for science is appropriate a calmer inclination, an inclination remaining ever patient—love is impatient; an inclination impartially estimating the value of a given manifold—love is partial; finally, the sciences require an inclination which does not allow predilections to check its progress, because the value of knowledge in very many cases actually consists in the accumulated amount, where isolated data would be useless, as in the case of isolated philological or historical items, isolated mathematical propositions, etc. To conclude, one must

ever feel himself master of the knowledge at the midst of which he is placed ; if to any part of his knowledge he takes up the attitude of the lover toward his fair one, he becomes a mystic or a pedant.

But, to continue, what purpose is this especial love to serve? Mr. Zippel has answered plainly enough. It is to cause a permanent apprehension of the subjects of knowledge, so as to give a man his bearings among them. Multiplicity, says Mr. Zippel, brings about confusion. But what if I should retort, And simplicity exhaustion? I need not enter into the depths of speculative psychology; I need only appeal to the commonest experience, to remind you that any, even the most agreeable, object can occupy us only for a while, and that everything that lasts too long is thereby spoiled for us. A sermon of an hour and a half is sure to become oppressive even to the religiously inclined, and a drama of five hours to a devotee of art. To the ordinary man such a duration will make either a sermon or a play a matter of positive repugnance. More so in the case of youthful and much more so in the case of children's souls. They demand change. One kind of interest, albeit the highest, can not fill their minds. There is, then, such a thing as too little, just as there is such a thing as too much. Midway lies a point of greatest vantage which is to be searched out.

Add to this that such especial love is apt to become determinate the latest in the very best minds. As long as I have been a university teacher, during a dozen years, I have constantly noticed in the most eminent among the students, men twenty or more than twenty years old, the painful effort it costs them to let go a number of subjects which they still desire to include in their work. On the other hand, I have known one-sided *savants* who, sated with the business of their specialty, gave themselves up to

trifles or even to trivialities—e. g., bee culture, town news, clubs, etc. But the love for the sciences appears in right good health where it is many-sided. The view of the intimate connection of all the sciences and of the aid they lend each other strengthens the charm of each.

Hence in the demand I address to pupils and to schools, I insist not on especial love but on equilibrating, many-sided interest. But I am at one immediately with Mr. Zippel the moment that in the place of interest there is substituted a mere learning, working, reciting, filling full of copy books, making of translations, etc. I unite with Mr. Zippel promptly the moment I notice anywhere the machinelike diligence of teachers and pupils tormenting each other simply that both may be able to say that they have done their duty. By this method, the teachers in reality do not do their duty; their occupations are not of a sort that can be despatched and done with. Where the joyous diligence of the scholars does not proclaim everywhere that they are fond of working, that has not been done which ought to have been done, even though the examinations for promotion and graduation furnish forth the most brilliant results.

But I must add a word concerning the most exquisite, the most delicate natures among the scholars. These ever have certain secret resting-points for their feeling and thought. They have an inner home whence that is likely to proceed, in far later years, which they are indeed to be and to effect. An instruction which does not at all, not even mediately, touch upon this point does violence to them. They, under it, are poor students. Now, what is to be done in the case? Above all, it is essential to discover these points of rest or these centres of gravity, or, if you prefer, these axes about which the mind revolves, in order to enable us to heed them and to have reference to

them. But only many-sided instruction can discover them. For these points of rest are as various as the genius dwelling in them. Sometimes they are religious, sometimes speculative, sometimes economic, sometimes military, sometimes—but who can enumerate all? Only a many-sidedly educative school can meet the very requirement in whose favour the simplified instruction is demanded, namely, that the inner home of the mind be sufficiently respected.

But while the possible resting-points of mind are manifold, that which ought to be the resting-point of all is simple—religion. Upon this point, however, Mr. Zippel, it seems to me, has been misled into an erroneous conclusion. He says that schools assign to religious instruction far too inferior a place—which means, I presume, far too little time—treating it as a subordinate branch. But it does not follow that what one assigns less time to, one holds to be a subordinate branch and desires to see treated as such. Suppose a man wishes to determine the amount of space for the various implements and possessions in his house according to value. Fancy what an amount the jewellery must occupy! But the precious stones would refuse to fill so much space, it being in kind for them to concentrate their preciousness into very little space. Not otherwise can I judge of religion. I know and acknowledge that it must constitute the deepest foundation and one of the earliest beginnings of human, ay, and of childhood education; without which all else is vain. I do not say this to-day for the first time; I have said it in the first of my pedagogical works, and, unless I am mistaken, plainly and emphatically enough. But a religious instruction expanding into a great number of regular lessons makes me afraid; exactly as I become alarmed at a prolix confession of faith, which prescribes in many articles the method and ways by which the human heart is to approach the

Most High; and since a considerable period I have been uneasy at the recommendations of religion such as are fashionable to-day, and which quite obviously have originated from the misfortunes and the miseries of recent years. These miseries, it seems, cause everything to be forgotten that knowledge of human nature, church history, and the history of philosophy unitedly teach; to wit, that every Jacob's ladder, whose rounds are exactly counted and are methodically to be climbed one after another, is unfit to satisfy the universal need for religion; that the truly religious people often have extremely few articles of faith, and that they who have investigated most keenly testify that what we know of religion and consequently can, in the strict sense, teach and learn, contracts very narrowly into a few extremely simple grounds in support of a reasonable faith. Such is my result also, although the so-called Nature philosophy of to-day teaches differently. Hence, it is my opinion that the younger children must be aided by moderately extensive instruction in order that they may apprehend the right concept of God and learn to seek him in Nature; that to the growing youth—whose instruction in Christian doctrine falls, of course, especially within the competence of the clergy of each denomination —must at school be communicated some knowledge of church polity and the different dogmas; but that otherwise we ought to have not so much detailed instruction but rather devotional exercises conducted in a Christian spirit, among which the Sunday sermons, if they are good and within the comprehension of youth, will be principal. But the intensive excellence of the instruction must in all lesson periods devoted to religion correspond to the intensive importance of the subject; and hence the question how good and how profoundly effective is the religious instruction communicated in a school, how perfectly pre-

pared does the teacher appear in every lesson, is not a secondary but a main question in evaluing the school. In addition, we may choose for the purpose those hours in which the students are best disposed for it, use the co-operation of harmonic song, and reprimand tenfold as severely as in other hours any, the least, disorder occurring during these periods. Thus much on this point, which to exhaust in this discussion is an idea which could not enter my mind.

Of the impracticability of the notion of having only one of the ancient languages learned, of the absolute impossibility of caring incidentally for all the other studies —for mathematics and natural science, especially—through philology, we have spoken recently. Mathematics and ancient languages will always necessarily remain the two trunk lines of instruction. The natural sciences in the main lean upon the former, history and the whole of æsthetic culture on the latter.

The proposal for self-teaching from books we also discussed recently at some length. The grounds on which Mr. Zippel declares for reading instead of oral instruction are the very same—very respectable and weighty in themselves—as those alleged for simplifying instruction. The slavery of the attention imposed on students by six periods of instruction is to cease; the freedom of taking up and laying down a book according to inclination and wish recommends reading and self-instruction. The latter we have but lately admitted to be valid for certain individuals, and surely oral instruction for the others will be shaped all the better for its purpose, if in the upper classes, where alone this measure is applicable, such of the students are segregated as are too advanced for the others, or, being superior to them in free self-activity, prefer to read

rather than to hear. But setting this aside, two replies remain which though, it is true, not abrogating Mr. Zippel's proposition, yet limit it. First, the oral instruction ought to be of such quality as not painfully to strain the attention of the student, save momentarily in exceptionally difficult subjects. Especially, it must never be without immediate interest and natural connection. Neither should it, as a rule, consist of a continuous speech, like a university lecture, but appeal to the students to enter into the discussion. By this means, some freedom is given to their accidental associations of ideas. Second, the discipline of the attention is itself very valuable. Of our university students we, after all, must demand that they, hour after hour, shall hear and understand, or we could never finish our courses; and we can be of use only to those who are able to follow us for an hour with sufficiently flexible attention. As a concluding argument, what sort of reading would even the books themselves receive from persons of such brittle attention as to be obliged to lay even a well-written book aside every half hour?

CHAPTER VI.

It is time to sum up results. We began by reading Herbart's Introductory Lecture to Students in Pedagogy. We connected his system with Pestalozzi's by reading his essay on Pestalozzi's Latest Work: How Gertrude taught her Children. We then rose to Herbart's Point of View for Judging the Pestalozzian Method of Instruction. We perfected our initiation into his thought by obtaining his views on Pedagogical Discussions, and by the perusal of a paper containing his observations on an essay which was read before a pedagogical society.

We are thus fully prepared for entrance into his concep-

tion of pedagogy as a whole. This is laid down in the essay On the Æsthetic Presentation of the Universe as the Chief Office of Education, the paper which Herbart mentioned in connection with the essay on Pestalozzi which we presented in Chapter II. It is best at once to define the expression, Presentation of the Universe, in Herbart's own words. "Experience, human converse, and instruction taken all together constitute the Presentation of the Universe."

The essay appeared in 1804 as an appendix to the second edition of Herbart's A B C of Sense-Perception. On the title-page of that edition, where notice is taken of this addition, it is defined as a Disquisition in General Pedagogy. This describes its character exactly. It is not in definition and the time of its appearance only that this essay is a forerunner of Herbart's chief systematic work, the General Pedagogy. It is so in content. It constitutes, so to speak, an abstract, brief but compendious, of those main positions which are systematically unfolded in the latter work. For this reason it not only furnishes, what Herbart originally intended that it should, a good means for appreciating the value of the A B C of Sense-Perception, but it constitutes also the best preparation for reading the General Pedagogy in case the reader should desire to go beyond the writings of Herbart immediately applicable to common-school work and into the general scope of Herbartian pedagogy.

But the Æsthetic Presentation must be studied very thoroughly. It is one of those writings which in order to yield up all that they contain of mental nutriment require, to use an expression of Herbart's, to be read through from beginning to end and then back again from the end to the beginning. We may save the reader, perhaps, the latter half of the journey by setting forth the system of thought in—

A Retrogressive Analysis.—Herbart intended, as we have said, that this essay should help the A B C of Sense-Perception to its proper place of appreciation. Sense-perception on the part of the student supplies, of course, the first elements of knowledge. But equally, of course, the educator, who, in the Baconian phrase should be the minister and

interpreter of Nature, will arrange the sense-perceptions of the child in, for example, object lessons exactly as he will any other work—namely, according to his view of the general purpose of instruction.

The question then is, What is the view that he should take? In the first place, it will be conceded that the object of learning is doing, and that before we can act properly we must have properly learned. This we can not do except by attention, by devotion to the subject in hand. It all comes to the accurate apperception of the objective data. In the second place, the child seeks for the laws that bind these data together. These laws may at first be of extremely crude empiricalness; the child may not even know the term laws; but that is what it seeks. But the data as well as the laws are manifested in time and space; in other words, in mathematics. It would have to depart out of time and space not to know them. Accordingly, mathematics is in point of fact the mental basis in object teaching, if we must conserve the very mischievous term, object teaching; mischievous, because, emphasizing the means instead of the end, it has in many schools proved either uninteresting or dissipative of the most productive element in first year primary work. We prefer to employ the appropriate term for emphasizing the activity of the child—sense-perception.

Mathematics, then, is the mental basis for apperceiving both data and laws. Such intuition is important not only for intellectual but also for moral education. The pedagogist familiar with the moral results of manual training, as well as the scientist who has observed the effect of science study on character, will bear Herbart out in the affirmation, "Man soon looks on himself as Nature, if but once he have learned to know Nature. . . . But no one is inclined to think himself into the strict lawfulness of Nature to whom the strict discipline of mathematics has not been imparted, together with her enlightenment."

That our unpedagogic text-books prevent imparting such mathematics to our children is no argument against Herbart. It is an excellent argument for reading his A B C of Sense-

Perception in order to learn how children can learn trigonometry as easily as botany or history. Contempt of science not only is—as Herbert Spencer stanchly and with eternal truth maintains—irreligious; it is, like all irreligion, radically immoral. But the psychological basis of all science for human beings who must live in time and space is mathematics. To return, however, to the moral question.

Obviously, we must know what is right in order to do what is right. But this is not all. There must be the motive power of desire, of impulse, of will, which is based on the whole range of altruistic feelings, on our love for our fellow-men, on that Godlike element in man which, to use the beautifully truthful language of Herbart, "seeks to join in with the care of Providence for our race." This strong motor power is not knowledge, it is sympathy.

Thus we take up again the two great strands of Occidental development, the realistic and the humanistic. But under the naturalistic change of view it is now the pupil whom we contemplate. It is not the branches of instruction that we consider as of intrinsic value in themselves. What effect have they upon the pupil? It is true that pedagogy must become a science. Where, then, do we place the pupil? At the node where these two indispensable elements of culture converge. We would train him in knowledge, first empirically, and next by the inductions of science, until he can grasp worldwide laws philosophically deduced, and conceive of this universe as a unity; and we would simultaneously train him in sympathy, first empirically as befits his tender age, but soon sociologically at the awakening, for instance, of such emotions as patriotism, until feeling gathers its holiest hue and obtains its most sacred intensity in the contemplation of the dependence of the whole universe on Him in whom knowledge and love are one.

This union of the two elements of Occidental education exempts him equally from the heathenish concept of a dark, lawless fate, peculiar to the ancients though imitated in modern literature, and from the intrinsically atheistic notion of transcendental liberty that resulted from Fichte's

philosophy of the Ego, and which Herbart is never tired of protesting against as destructive to the very idea of pedagogy, which must assume at least that the pupil is modifiable by the educator.

On the contrary, our pupil, while, as we have seen, "joining in with the care of Providence for our race," realizes also his limitations. He recognises the world as a "system of forces" ordered by Providence, to begin with, for generic not individualistic accommodations. This opposition, this individuality of position is accountable for the state of need in which man finds himself. It fixes not the vague fate of the ancients, but a definite destiny. To fit into this place of his, man must take the measure of all helpful forces, that according to such measure he may use them and regulate his own steps. For, to be sure, the ethical idea is the voice of God to our race; but for that very reason it marks the goal, it does not helpfully measure the steps of the individual to that goal. That is left to us. Oversight of this distinction is responsible for the discussions on the permission of evil in this world. "To the actions of man is left the theodicy," is Herbart's conclusion, a conclusion both severe and sublime scientifically and morally.

Thus, then, we find scientific cognition recognised as the indispensable servitor, if not, indeed, as the basis of moral ideas. But Herbart is far from substituting the former for the latter. We have already said that the pupil must be carried beyond the personal ethics that prevail in early childhood. Herbart is far too serious a pedagogical scientist to have a trace of respect for the negative moral education which cowardly shirks its most sacred duty in pretending not to know that the child sees the evil which no one with his eyes open can help seeing.

Herbart as resolutely as in intellectual education seeks for the right sequence of instruction. He finds it in the parallelism of culture epochs of the race with culture epochs in the individual. The Homeric poems for childhood, historic writers for the growing boy, modern history for the youth approaching maturity, Platonic exclusiveness of aught,

however artistic, that might injure the moral picture of the world which is to be unrolled step by step like the natural picture—such is Herbart's plan put into one sentence and crumpled somewhat in the packing. But understanding that Homer occupies the same position as the initial point in the education of the sympathetic nature that the A B C of Sense-Perception, whose mathematical nature will become perfectly plain as the reader studies it, occupies as the initial point in the cognitive education, another approach has perhaps been made to understanding Herbart's insistence on Greek literature and mathematics, which, as we have said, has so often failed of being exactly apprehended.

We need not again call attention to Herbart's position on religious education, so strikingly illustrated in the preceding observations. The reader will not only find it reaffirmed here, but put in its proper setting in the Herbartian scheme. Herbart does not attempt, it will be seen, to suggest an ethical novelty. On the contrary, by him are laid more firmly and broadly than by any educational thinker the base for "the old and genuine morality." Under this aspect both reading choice authors and training the senses to apperceive art and Nature are comprehended. Herbart is absolutely upright. We have seen him protest against the fanatical systematization that strives to reduce everything under one principle, that some metaphysicians are possessed with. Here he abstains from enunciating any one principle in ethics from which all the rest are to be deduced. His procedure is thus opposite to the Kantian method of starting with a categorical imperative. Such deductiveness is contrary to the realistic metaphysics of Herbart as it is to the extremely vital and truly explanatory psychology of Herbart, so very different from the abstract and nonquickening psychologies of older date and more metaphysical cast. Whatever be the merits of the question metaphysically, no educator can do anything with such ethics or such psychologies.

Every practical teacher will agree with Herbart when he makes substantially the following argument: Let us admit

that goodness of will, virtue, moral cognition, the succession of subordinate laws, and the subsumption of every individual choice under those laws are all deducible from his supreme ethical concept by the moral philosopher, and that he can require it all as a manifestation of transcendental freedom of the will. Of what possible use is it to the teacher to look at the matter that way? We are not so happy as to find those things in our schoolrooms. So far as the educator is concerned, morality is a natural phenomenon, an occurrence in the mind of his pupil. The latter shows it occasionally at certain moments not by any means in its whole comprehensive entirety, but in small parts. There are isolated acts of the will having a certain strength of goodness at this moment; at the next possibly less, or none whatever, or more. Ay, and sometimes we find that this will element has taken a turn; that it is no longer positive for the good; that it is negative, is evil.

These observations are perfectly patent to every wide-awake teacher if the children display their emotions openly. And yet this phenomenon ought not to be partial and evanescent; it ought to be comprehensive, it ought to be lasting. Morality ought to apperceive all the other happenings, thoughts, fancyings, inclinations, desires; it ought to convert them into parts of itself. Morality ought to act with the full mental force of the pupil. All the presentations that make up the pupil's consciousness should be so harmonized.

We repeat a remark previously made: Educative instruction must be construction. What will determine such construction into character? Supposing that we have been careful, as we have seen Herbart to be, to avoid reducing to unity irreducible judgments, which includes being careful to admit collisions where they actually exist; supposing, in other words, that the pupil has from the first acquired each simplest judgment in its clear, precise form and without foreign admixture; how will the regulation of life result from their harmonious construction?

The educator assumes that he can determine that con-

struction. Psychic laws act as surely in his pupil's soul as physical laws in his pupil's body and in all other bodies. The physical scientist asks Nature questions and utilizes her answers in the material arts. The psychic scientist frames from the answers of Nature the art of education. The powers are given. They are necessarily true to their own nature. The psychic scientist puts them into such relations as will result in moral elevation; that, at least, he considers the master problem in the whole of education.

To cause the pupil to find his true self as rejecting that which is evil—that or nothing is character-building. This alone is that moral freedom which the educator seeks to bring about and to strengthen. Morality, then, is essentially related to an indefinite multiplicity of volitional acts. It is for this reason that all the deductive metaphysical systems of morality are so unsatisfactory. The analytical part is plain enough. The concept of morality, or, to put it psychologically, the concept of goodness of will, immediately suggests the two correlative concepts of obedience and command, and it is conceded on all hands that true morality is not something imposed from without, but that the truly moral man is self-commanded. But when we get beyond this concept of mere morality, when granting this definition of morality as imposed not by bribery or threats of punishment or any other external means, but by self, and as consisting in the relation, putting it again psychologically, of an obeying will to a commanding will, when we ask for something additional that will carry us beyond mere definition, when we beg synthetically to be informed what it is that is so commanded and accepted as a command, we strike vacant space.

Of course. an attempt is made to fill it. We have mentioned Kant's Categorical Imperative, which substitutes in the place of any real content the mere form of the law, to wit, its universality, and Herbart adds to it several other illustrations. But the trouble is obvious. As we have conceded the definition, it is agreed by both sides that the concept of morality is known. If that concept contained only one

single object, we should, of course, knowing the concept know the object. When two people know the concept of a horse, they know a horse. But in this instance the only thing that corresponds to moral obedience is the general concept of a volitional state assuming the force of command as against all individual desires and accidental longings.

That general concept, as we have seen, embraces a vast multiplicity of presentations. These must enter into such a construction as to be comprehended under the one general concept of morality, the commanding concept to which alone we can vow fealty and which alone can induce that concentrated attention, self-criticism, and humility which constitutes the surest guarantee of our moral constancy and growth. Casting out all foreign elements, this construction must progressively differentiate between morality and immorality, constantly increasing the governing will, proportionately diminishing the volitions still requiring to be subjugated, until we can well conceive that an indefinite continuance of the constructive process would approach us infinitely to moral perfection.

Such construction is not a logical one. It is possible for a person to be a logical expert in moral casuistry and yet a consummate rascal. Neither is the necessity implied in a moral law of the same kind as that which scientific theorists set forth in the laws of Nature. Everybody knows the difference between *ought* and *must*. By these exclusions we find that the only remaining necessity is the æsthetic necessity, that necessity which judges immediately and without any proof.

Herbart does not, of course, mean to identify artistic taste and moral consciousness. The latter originates, as we have seen, in the relation of a commanding will to an obeying will. The former of these two, the conscience, as it is summarily called as a sort of collective noun, we have seen to be a complex apperception mass aboriginally compounded of innumerable presentations. He is now inquiring by what constructive process these presentations are built into that apperception mass. In other words, he is inquiring

into conscience-building or character-building. Of course, when the conscience as commanding will enters into relation with the isolated impulse there is moral necessity. But the problem of the teacher lies further back. The moral educator constantly asks himself, How can I be a conscience-maker? Herbart's answer is, By availing yourself of the inherent æsthetic necessity in presentations, arising inevitably in the simplest æsthetic relations, you will have self determined by self and yet no self-will, but an inner necessity, neither logical nor material, but causing men to do right and visiting them if they do not with compunction. To be sure, in art your pupil may escape the necessity. He may give up the particular art that plagues him. But no one can give up the art of being a man or a woman. The use of the word art in this connection is not an attempt at cleverness or simile. The cases are psychologically parallel. In the great art of right living, as in the little art of rightly playing the violin, we are guided, restrained, and admonished by an overriding apperception mass, and we need much detailed knowledge of our position in the external world, a subject on which we have already expatiated.

In both cases the overruling apperception mass is built up out of simple æsthetic judgments of repugnance and agreement. In neither case does the inherent necessity always demand the existence of something. But it perennially affirms that if something does exist, it should exist in harmony with it. In both cases one may, whether wilfully or blunderingly, fall short of obedience and take the consequences in compunction. In neither case is the coercion either logical or material. In both cases we judge others like ourselves, and, what is more, in both cases we require that they shall so judge themselves.

As the artists' faulty apperception explains much blundering in art, the accidental picking up of daily observations explains why there is so little solidity and unity of character. It is this accident which is not to remain accident if education is good for anything. It is morally obligatory upon the educator to determine the pupil's ap-

perception early and strongly by an æsthetic presentation of the universe. To be sure, as has been intimated, we purpose to allow the truth of history and the transfiguration light of poetry to clarify by contrast and to vivify by variegated hues that presentation. But we insist also for the purposes of that construction on the correctness of drawing, on science, on knowledge, and as the basis of it all on exact, perfectly truthful sense-perception.

And now we come to the pedagogical application of all this psychology. If the commanding element in the relation that we describe as moral necessity is will, so also is the obeying element. This latter will is not to be suppressed. No sound educator will suppress aught of will any more than he will suppress aught of intellect. But it is to be educated. What does that mean? It is to change its direction. Yet we do not withdraw the objects. We do not forbid the objects. On the contrary, by making the student know and think much and by attending to the proper association of his presentations we cause him also to desire much and to associate properly his desires. Even well-bred children possess so much practical freedom of the will as suffices to inhibit desire for the time without any great effort. A well-educated man, having more and better associated presentative knowledge, possesses such practical freedom to a greater extent.

Now, the only remaining question is whether egotism or moral sentiment, or, as Herbart calls the latter by a term unfamiliar to English ears and yet beautiful and suggestive, the Practical Reason, shall obtain mastery of this power of self-direction and use it as its instrument. In the former case the result is worldly shrewdness, in the latter morality. Here, then, is the crisis in educational art: for what shall it profit a man to have mastered all the world, including himself, if he lose his own soul, and what shall it profit the state to have educated clever rascals? And this crisis is a perennial, not a momentary one. On the one hand, many desires must be awakened; on the other, none must be permitted unbridled to rush for its object. This balance must

be achieved by a training presenting itself as an impersonal necessity, sweetened with many a kindness and guided by the consummate ability of the educator to expunge in children self-will without any injury to their cheerfulness. Thus he will foster a many-sided, equilibrating interest, prevent a coarse desire from acting itself out, and by thus emphasizing its force in consciousness becoming will, while yet leaving himself free to put in action any well-thought-out group of motives, thus teaching practical reason its force.

To be sure, Æsthetic Presentation of the Universe is a wider term than the narrowest definitions of morality might seem to require. But it is nowise safe to draw a line. As it has been said that there is but one virtue, the only safe way is to say that there is but one æsthesis. It is by this magnificently psychological conception of education that Herbart has rid us of that chaotic condition of educational thought which consisted in assuming as many different educations as men had different purposes, and which still where it prevails renders systematic unity in work and theory impossible. That chaos had, of course, among its elements one element called moral education. It even admitted that it was the most important. But the admission remained verbal, because moral education was set off in a separate compartment like the rest. That it must comprise the whole human being never dawned on these unscientific pedagogists. The reason was, that they were shackled by the old and inane psychology with its verbal division of the mind into equally separate compartments termed faculties.

The psychology of Herbart conceives of the life of the mind as infinitely varying in its presentations. Under the latter conception all of mind subserves or conflicts with morality. That upon the deficiency of which life is wrecked, that upon the perfection of which life is rested, moral character and conscience, is to be upbuilded of the whole mental nature. Thus considered, all else are but the necessary prerequisites, the presuppositions, the conditions for the actualization of morality. Thus considered, pedagogy is a unit, is

a science. And it is not its least merit that its existence as a science is rested by the great philosopher upon the bed-rock of morality.

Considering pedagogy as a science, it may truthfully be said that earlier pedagogists could have no system of pedagogy, though they might have disjointed pedagogic truths of the highest importance. They could have no educative instruction as a unit, though they might have very excellent didactic systems. Pedagogy as a unit, as a science, became possible when Herbart made the discovery which, as we have said, corresponds in this branch of human endeavour to Newton's discovery of gravitation in astronomy. This single unifying principle he enunciated in the sublime sentence with which his Æsthetic Presentation of the Universe opens.

The Æsthetic Presentation of the Universe the Chief Office of Education.

The one problem, the whole problem, of education may be comprised in a single concept—morality.

It would be equally possible and permissible to assume as many problems in education as purposes are allowed to human beings. But then there would be as many pedagogical investigations as problems; these investigations would be carried on without correlation, and in the isolated measures of the educator it would not appear either how they must limit one another or how they can promote one another. We should find ourselves much too poor in means if we tried to attain every individual end directly; and, again, no matter what it might be that we intended to produce but singly, it might possibly occur tenfold through concurrent and subsequent effects which were not intended and not allowed for, so as to put all the parts of the work out of their proper relation. This

method of considering the subject is thus unserviceable as a starting point for pedagogical investigations. If it is to be possible to think out the office of pedagogy as a single whole with thoroughgoing correctness, and to execute it systematically, it must be possible first to apprehend the problem of education as a single problem.

As the highest purpose of man, and consequently of education, we universally recognise morality. He who should deny this could not really know what morality is; at least, he would have no right to take part in this discussion. But in order to set up morality as the whole purpose of man and of education, an enlargement of the concept of morality is required, a demonstration of its presuppositions considered as needful conditions of its actual possibility.

Goodness of will—the steady determination to think of one's self as an individual under the law that obligates universally—is the commonest and, rightly, the first thought suggested by the word morality. When in thought we add the force, the resistance, with which a human being maintains the goodness of his will against those emotions that work against it, morality—at first only a quality, a determination of the will—becomes for us virtue; it becomes strength, and action, and effectiveness of the will that is so determined. Distinct again from both is that which pertains to legality, correct knowledge of the moral law. And, again, different from the knowledge of the universal law, and even from the knowledge of the rules of duty usual and recognised in common life, is the exact judgment of what in special cases, in distinct moments, in the immediate touch of man and destiny, is to be done, to be chosen, to be avoided, so as to secure the best, that which is truly and only good. Philosophy finds all this in the concept immediately, and of man it expects

it and demands it just as immediately, as a manifestation of freedom.

But, as thus set forth, can an educator make anything of this mode of presentation?

Even supposing nothing to be under discussion but moral culture in the narrowest sense, even divesting it as far as ever one thinks possible of all that is scientific, of all the exercises, of all the fortifications of spiritual and physical energy, laying them aside for further consideration, is that which now, taking up only the concept of morality, offers itself to the philosopher a datum also for the educator? Does he, too, find that goodness of will, so as to be only obliged to direct it against the inclinations and to point it to the right objects by his lectures on morality? Does there flow for him, also, that intelligible source? May he also confidently derive from Heaven that stream whose well-springs he knows not? Indeed, for him who adheres to our more recent systems nothing is more consistent than calmly to expect that the radical good, or possibly the radical evil, will perhaps quite spontaneously manifest itself in his pupil. Nothing more consistent than quietly to respect the freedom which he must, of course, presuppose in his pupil as a human being, abstaining simply from interfering with it by any misapplied endeavour—which, by the way, tempts one to ask, Then, after all, freedom is subject to interference? Thus he abandons the most important part of his office, and in the last resort confines all his care to the mere presentation of items of information. It is a fact that something similar has been put forth as a serious assertion by an adherent of those systems.

But we must not be so precise in the application of these theories. Under the weight of such consistency they themselves would have broken down in the moment

of origin. We may hope that the first transcendental philosopher interested in education will find means to show for education, too, a suitable basis. The postulate—education must be possible—will first be endowed with a valid title. This granted, there is plenty of room in the world of sense, and for any one who has any business in it the realistic view holds. As freedom by enunciating the moral law may betray its presence, as though it were a cause in the realm of phenomena, so the world of sense ordered by the educator will be allowed to seem as though it influenced the freedom of the pupil. That suffices; we now have our field. It is true we have not yet the rules of procedure. But let the educator make a beginning. Let him invent them. Do not doubt, the transcendental philosopher will afterward know how to derive them from his system. As far as the educator is concerned, morality is an occurrence, something happening in Nature, which, as we may assume, has already shown itself in the soul of his pupil in occasional moments and in small part accidentally, but which must occur to its entire extent, which must last, and which must take up into itself and convert into parts of itself all other occurrences, thoughts, phantasies, inclinations, and desires. In this completeness this natural occurrence ought to happen in the pupil with the whole quantity of spiritual energy; in the incomplete form in which it does happen, the goodness of will has always —or, rather, every single good volition is always—a definite quantity of activity, a definite part of the whole, present in this particular determination and magnitude only during this determinate moment. But in time the quantity grows, decreases, vanishes, becomes negative, like a curve line, and grows again. All this is open to observation to the extent that the pupil frankly manifests himself.

Under all the determinations with which it happens it happens necessarily, being the unfailing result of certain mental causes. It happens just as necessarily as any result in the physical world, though it does not in any sense happen according to material laws, as those of gravitation, impact, etc., which have not the slightest similarity to the laws of mental effect. Like the astronomer, the educator imposes upon himself the attempt, by a right questioning of Nature and by accurate conclusions sufficiently prolonged, at last to search out from the course of the phenomena presenting themselves to him the laws which govern them. This implies the discovery of how that course can be modified by purpose and design. Now, this realistic view does not admit of the slightest admixture of the idealistic view. Not the faintest breath of transcendental freedom may blow through any cranny into the domain of the educator. What in the world can he do with the lawless miracles of a supernatural essence whose help he can not count on and against whose disturbing influences he can use neither foresight nor precautions? Is he to furnish occasions, to remove hinderances? After all, then, that absolute faculty was hindered? After all, are there, then, occasions for it outside of its own purely original commencing? After all, then, the intelligible, again, is immeshed in the mechanism of the objects of Nature? Let us hope the philosophers will reflect better upon their own concept. Besides, transcendental freedom neither should nor can be found in consciousness, as if it were an internal phenomenon. But that freedom of choice which we all find within ourselves, which we honour as the fairest phenomenon of ourselves, and which we would like to emphasize among all the phenomena of self, is exactly what the educator strives to effect and retain.

Causing the pupil's discovery of himself as an elector of good and rejecter of evil—this or nothing is what we mean by forming the character. Doubtless, this elevation into self-conscious personality must take place in the mind of the pupil himself, and must be accomplished by his own activity. It would be nonsense for the educator to desire strictly to create the real essence of the force for the purpose, and to infuse it into another soul. But to set the existent force, which is necessarily constant to its own nature, into such a position as will make it work out that elevation unfailingly and unerringly is what the educator must deem possible. To get at it, to fix upon it, to fathom it, to bring it in, and to guide it on, he must regard as the great object of all his endeavours.

It becomes necessary now to subject the concept of morality, which we are obliged here to consider as known and a datum, to a keener philosophical scrutiny. We shall begin by mere analysis. As we continue we shall come to necessary synthesis by showing up the presupposition to which the concept essentially relates, while yet unable to consider it comprised in the content of the concept. This form of investigation is of very general use, though here, we admit, it can not be exemplified with absolute sharpness and accuracy.

Obedience is the first predicate of the good will. Over against obedience there must be a command, or at least something that can appear as a command. A command has for its object something commanded. But not every obedience to any haphazard command is ethical. He who obeys must have examined, elected, and valued the command—that is, he must himself have elevated it into the position of a command to himself. The moral man commands himself. What does he command himself? Here the embarrassment is universal! Kant, than whom none

was more sensitive to this embarrassment, after much hesitation, quite hurriedly ends by slipping the form of the command, its universality, which distinguishes it from momentary volitions, into the place of the content.* Others slip into this place their theoretical concepts—approach to the Deity, to the pure Ego, to the Absolute, nay, even the customs and laws of the country, or even the useful or the agreeable.† The unprejudiced investigator will recognise the empty place to be empty. He argues: We all know the concept of morality; if the concept contained a single definite object of command, we should know that, too, together with the concept. Therefore, it does not contain a single definite object. Yet it is related to a presupposed command—that is, a presupposed willing, for command itself is will. This willing must be original and primary; obedience follows after. Now, if this original willing is not definite but yet actual, it is obvious that it is indefinitely manifold. Herein lies the reason why it is not deducible from obedience. To obedience there corresponds as command only the general concept: There exists such a willing as stands in the form of a command over against all inclinations and individual, accidental desires.

Before we seek for the characteristics of those mental acts which here, as opposed to the obeying will, appear as the commanding will, we have to make two observations. In the first place, these acts considered by themselves can be nothing ethical in the strict sense. They exist antecedently and independently before entering into a com-

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* See the Foundations of the Metaphysics of Morality, p. 51.

† We may here compare a passage in the sixth book of Plato's Republic: "To the crowd, pleasure seems the good; to the more educated, good sense." Plato goes on to censure the circle: "They can not tell which sense, but are of necessity driven in the end to say, The sense of what is good."

manding relation as against the inclinations. Only as far as they become one member in this relation do they pertain to morality. What is original in the commanding will must be looked for in an altogether different sphere. In the second place, as far as these indefinitely manifold acts are motives of obedience, they must be so construed as to be capable of being comprehended under that general concept, to which that general and single pledge of fidelity applies, together with single and constant attention, self-criticism, and humility, which constitute the crown of ethicalness. The quality of the construction must be such as to expel every foreign element. By it severity of opposition must be introduced between the worthy and good on the one hand, the unworthy and the evil on the other. And by it the loud, incisively forceful language of the moral imperatives must originate. For anterior to the relation of the reason to the inclination this is all inconceivable. Such a construction can not be merely logical. It can not be learned from a well-classified ethics. The latter cools the will; it does not urge it on. On the contrary, there is needed a construction which is partly poetical, partly pragmatical. But it is time to look for the elements which are to be construed.

It would be vain to lay upon the desires the injunction of obedience were we afterward to transmute reason into a desire. Kant's proposition is eternally true: No ethical principle must ask the actuality of an object. But from this what follows? Nothing but this, that originally reason is not will at all, since will that wills naught would be a self-contradiction. The reason apperceives, and after completing the apperception it judges. It views and pronounces the verdict. Then the eye is turned to look further. We shall find this confirmed by taking up our previous thread. He who obeys attributes validity to the

command—that is, he originates it, at least, as a command. Now, how in the act must he necessarily appear to himself, as establishing an imperative or as finding an obvious necessity? Must he needs intend self-assertion as the lord and master, the proprietor, so to speak, of his inner store of sense and life? Or would it perhaps be, if not truer, in any event safer for the correctness of his judgment if he only strove to fathom, let us say, the will of a perfect reason outside of himself? He must not appear to himself as establishing the imperative, since the primary principle of morality, obedience, is annihilated, and one arbitrary volition put in the place of another as soon as will, under any interpretation of the term, shows itself as the ground of command. The ethical man is humble through and through; this acquaintance with the concept of morality we presupposed.

He therefore appears to himself as finding a necessity. Or perhaps he does not appear to himself at all; it is conceivable that he might find the necessity without directing his gaze upon himself. This question will answer itself more accurately a little further on. For the present the question is, What necessity is found? Not a theoretical necessity, for the difference between *ought* and *must* is well known, and to honour a command does not mean to yield to the unalterable. Neither, for the same reason, is the necessity logical, for a logical necessity considered by itself is also a *must;* and, again, it relates to a higher principle, and therefore only postpones the question how and why the latter principle be necessary. Hence, this necessity is neither inferred nor learned, nor bestowed by experience, nor searched out through natural science. To this extent we confirm Kant as altogether in the right in setting in sharp opposition empirical details and pure reason. It is to be hoped we shall not be met by the answer,

"A moral necessity"! We showed but a moment ago that we are here quite without the domain of morality. The discussion is upon what is aboriginally necessary. This will perhaps become morally necessary, but only from the instant when it governs the obedience in opposition to the inclinations.

Only one of the known necessities is left—æsthetic necessity.

Its character is, to speak in none but absolute judgments, entirely without proof; but without, on the other hand, putting violence into its demand.* It has no respect to the affection, neither favouring nor contesting it. It originates at the complete presentation of its object. For different objects there are as many original judgments. There is no mutual reference in order to logical derivation. At most, after eliminating all that is accidental, similar relations are found to recur in different objects, thus naturally producing similar judgments. As far, accordingly, as the simple æsthetic relations are known we have simple judgments upon them. They are the axioms of the arts and have absolutely independent authority. In this respect, music is eminent among the arts. It can enumerate definitely all its harmonic relations, and as definitely show their correct use. But if a teacher of thorough bass were asked for his proofs, he could only laugh at or pity the dull ear that had failed to apperceive. It is specially important that æsthetic judgments never demand the actuality of their object. But if the latter does exist and continues to exist, the judgment, too, persists, which indicates how it ought to be. By this persistence it becomes to a human being who can not es-

* The bond of reason is soft because it is golden, for reason is beautiful; but gentle, not violent. (Plato, Laws, i, 13.)

cape it at last an equivalent for severest necessitation. To the artist an offence against taste is a crime; of course, only to the extent that he intends being an artist. He is not barred from destroying his unsuccessful painting, from locking up the instrument which he can not master, or, in the last resort, from quitting art altogether.

Only from himself man can not part. Should he himself be the object of such judgments they would, by their quiet yet ever-audible speech, in time coerce him, exactly as they do the amateur who has set his heart upon becoming an artist. Add that a judgment of taste bursting forth from the midst of the sentiments is frequently, by the mode of its origination, felt with a degree of force which does not strictly lie in what it says. Happy if such an outburst conquer on the spot. In time it passes away; but the judgment remains. It is its slow pressure which man calls his conscience.

Finding, therefore, an originally practical and hence æsthetic necessity, the moral man bows his desires in order to obey. The desire, then, was one member in an æsthetic relation. The man who contemplates the latter directs his view upon himself to the extent that the desire exists in him which occurs in the relation that is judged. Doubtless, however, the æsthetic demand would remain exactly the same if the desire were entertained by another standing in the same relation. Thus we judge others, only still more easily than ourselves. The demand is valid, or should be so, at least, for the other person; we expect of him that he should himself find it to be so.

Wishing to become acquainted with those among the æsthetic judgments which are directed upon the will—that is, wishing to establish a practical philosophy—we must, above all, abandon completely the idea of a single highest moral law as the one dictum of pure reason, all

other moral rules being but applications. On the contrary, step by step, taking into consideration the simplest conceivable volitional relations which can result out of the direction of the will upon itself, other wills, and things, there would stand forth with immediate evidence for each of these relations an original, absolutely independent, æsthetic judgment of a quite peculiar quality. The judgments thus obtained would have to be construed afterward, so as to order life. It would be easy to do so if we had gained them at the first in their peculiar clearness, in their simplest and most precise determinations, unmixed with anything foreign, and undisfigured by the attempts of a false philosophy at reducing them to one another. The opposite procedure easily explains why it is difficult to erect out of the judgments caused accidentally and in a scattered fashion by daily life a firm system of moral convictions, from which the character might obtain solidity and unity. But if science had taken care in this construction for the correctness of the drawing, then the riches of life—partly transfigured through poesy, partly enforcing themselves as the truth of history—would help to present the picture, now as a whole, now as a part, illuminated by changeful colouring, emphasized now by this, now by that contrast.

But this pedagogical thought comes too early, although but a little so. For we are drawing near to the application of the general considerations. We only need glance back at moral obedience. How is it related to this system of practical reason? * What obeys is to be and to remain

* Let our psychologists not take it amiss that practical reason here partly coincides with æsthetic judgment, while, on the other hand, it knows absolutely nothing of any relationship with transcendental freedom. On the plan adopted by us it is, in fact, impossible to see how transcendental freedom could enter into practical phi-

will. But it is to change in part its direction.* Now, originally, all willing, desiring, demanding is directed upon objects.

Do not believe that these objects can, as it were, be moved about under the will directed upon them. One who wills but little is utterly disgusted when this little is denied him. It is only in the abstract that we can separate the will from its direction.

But he who knows and thinks much has many desires. And he whose ideas are well associated also associates his desires. To change the direction of desire means, in truth, to impede one desire—so, however, as to have another in readiness to emerge at once by its side. This is impossible except for a mind that is versatile and awakened in many directions. This is exactly the reason why it is easier for a man than for a child. But well-bred children even have been given, and by their very training have acquired, the freedom of inhibiting for the moment any desire without great difficulty—a freedom, by the way, which, taken by itself alone, has as yet nothing in common with morality. Nevertheless, it is at once obvious that all that still depends is, whether egotism or practical reason shall have dominion over it. In the former case it becomes worldly acuteness, in the latter morality.

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losophy. One might as well mix it up with the theory of music or sculpture. As for the consequences to be feared, let us for the present console ourselves by education. On the other hand, it must be extremely welcome to theoretical philosophy—as Kant, at least, intimated plainly enough—if it be no longer necessary for her, on her sister's account, to suffer that misconcept whose self-contradiction she would otherwise, I am sure, have confessed long ago, and was often on the road to confessing.

* In this connection compare the *σοφία*, *ἀνδρεῖα*, and *σωφρόσυνη* of Plato, especially as presented in the fourth book of the Republic.

Here, then, we have laid at once before our eyes the primary requisite of training. We must awaken much desire, but we must permit no single desire, unbridled, to rush for its object. It would seem as though an immeasurable supply of will were confined in an iron receptacle, opened only by Reason, where, when, and how it pleased. It will so seem if from the first the object's touch be as multiform as possible, while the bridle felt always is, if need be, effective enough to impress firmly upon the mind that the attainment of no object is to be counted upon unconditionally. It is, of course, well known that training best presents itself as impersonal necessity, and that it must be compensated by much love, by much undeserved obligingness. We make a general presupposition of the art of overcoming all that is called stubbornness in children, without injuring their cheerfulness.

Exactly as we must prevent crude desire from demonstrating its force by its own action, and thus becoming determinate will, thoughts rightly weighed must, whenever they emerge, be converted into activity and seconded until they attain their purpose. Reason, thus experiencing her power, takes courage to govern.

When we see a boy who, whether the thanks be due to art or to Nature and accident, tries many things, but readily forsakes what he finds foolish, while he firmly and vigorously puts through what he has well reflected on—a boy who is every way easily awakened, who by improper treatment is easily irritated, but who by the right word one can easily teach, turn, and shame—we rejoice at the sight and augur well of him. We term him free because we presume that, being open-eyed, he will readily find and apperceive the rational, while there is no lurking opposition in him which might silence or overpower judgment.

But perhaps we forget that it is still a question what

kind of a world the boy will find before him, will judge, and will practise dealing with.

Let this world be a rich and open cycle, full of manifold life! If so, he will scrutinize all its parts. What he can reach he will touch and move, in order to investigate its entire mobility. The remainder he will contemplate and will mentally transport himself thither. He will pronounce judgment upon men and their conduct, and compare ways of living and social positions as to their splendour, their profit, or their freedom from restraint. He will—in thought, at least—taste, imitate, choose. If a firm hold is got upon him by any such charm, he calculates, and is lost to genuine morality.

But, on the contrary, let nothing fetter him. Let his boyhood years elapse in the continued whirl of momentary joy. Only let him be sure of his physical strength, his health, his freedom from wants, and his inner firmness; only let him have gathered together by occasional observations a sum of keenly noted phenomena, so as not to feel a stranger among the things of the world. In the next place, let him become aware of the decorum which is required from a grown youth from his first entrance into society. Shy of making mistakes, wishing to learn; but as to the rest, quiet, without seeking or fearing aught, let him enter and look about. Concentrated vigilance will grasp all relations. The contrast between the ridiculous and the becoming will determine his judgments as easily as his conduct. Besides the becoming he will find what honours or shames, uprightness and good faith, falsehood and treachery. In the next place, if only his mind be truly imitative, he will from the first be full of sympathy, full of an inclination to enter into the sufferings and hopes of others. He will thus be predisposed for that reflection which recognises and values goodness, which is

the beauty of the soul. Out of these apperceptions he will establish for himself a law and a duty to follow that law; he can not do otherwise; he would be obliged to despise himself did he not follow it. Hence, he wills to follow it, and he has the power to do so; and again, with increased emphasis, you will term him free. Rightly so, and in the noblest sense of the word, no matter though you know ever so exactly how he came to be so, and could not but become so.

Whether he did not or did become so, and how far, depended on the psychological accident whether he became first immersed in the calculations of egotism or in the æsthetic apperception of the surrounding world. This accident must not remain accident. The educator's moral duty requires of him the courageous presupposition that it is in his power, if he set about it in the right way, to determine the pupil's apperception early and strongly enough by the æsthetic presentation of the universe, so that the free mental attitude shall receive its law not at the hands of worldly prudence, but from pure practical reflection. Such a presentation of the universe, of all the world that is known and of all the times that are known, in order to efface, if need be, the evil impressions of unfavourable surroundings, may justly be termed the chief office of education, for which that training which awakens and restrains desire would be but a necessary preparation.

The concept of the æsthetic apperception of the world is larger than that of the similar apperception of human desire. It is therefore larger than the immediate demands of morality. And it ought to be so. For although external objects are to us accidental, and although it is very important to reckon as much as possible among the accidentals, it is impossible for us to depart altogether out of the sphere of external things. Consequently, many and

various demands of taste arise, whose manner of demanding is fundamentally none other than that of æsthetic judgments of the will. Accordingly, their urgency is felt the more strongly the more closely the external is connected with ourselves. This accounts for the power with which external honour, decorum, social tone—in short, everything which pertains to ridding ourselves of crudeness—presents its claims among people whose culture has begun. It is said there is but one virtue. With almost equal correctness one might say, There is but one taste. He who violates it anywhere with cold reflectiveness is upon the road, if not to abandon that which is moral, at least rather to found it on alien principles emanating either from the striving after self-dependent greatness and welfare or from civil and religious prudence.

As to the arrangement of a general Æsthetic Presentation of the Universe, just one word, really a repetition of the preceding: Avoid reducing the judgments of taste one to another. And what comes to the same thing, avoid denying the existence of conflicts. A little further on we shall demand much and early reading of selected classical poets, and a preliminary exercise of the senses for apperceiving works of art of every description. The causal connections, even where the reasons are passed over, are easy of surmise.

In addition, only a few of the main traits of that presentation of the universe as far as it immediately concerns morality.

Everybody grants that even a child can not miss the simple fundamental judgments on volition,* considered

* One has occasion at the present time to protect one's self from the suspicion, as though one intended to invent a new ethics, and thus to mock the strict demands of the old and genuine morality. Therefore, we shall here allow the familiar names of righteousness,

not as formulæ but as judgments on individual cases, by very reason of their simplicity and absolute priority, provided only the opportunities are afforded to it by the surroundings. It is often said, and let us hope universally recognised, that a mother's tender care, the friendly seriousness of the father, the family ties, the order of the house must present themselves to the eye of childhood in all their purity and dignity, because the child judges only what it has observed; nay, because what it sees is to it the only thing possible and the pattern for its imitation.

Suppose this first condition to be fulfilled, or suppose it to be made up later on by the beneficent humanity of a not common teacher, how, from this point, does education proceed? It must leave the narrow circle. It shows weakness worthy of the severest censure if from fear of what lies outside it attempts any longer to confine to its nearest surroundings a child who has ended its apprenticeship and looks and strives further. Education must progress upward and downward. Upward there is one step and only one; there is nothing higher. Downward there are width and depth unlimited. In the former direction the supersensuous realm must open, for of all that is visible the family circle is itself the fairest and worthiest. But in the opposite direction lies reality. It partly with the obtrusiveness of sensuous clearness shows its own defects and its necessitousness. Partly it is the duty of education to disclose completely what the pupil does not see, and yet must see, in order to be able to lead the higher life of man.

Since contrasts emphasize each other, and the more so

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kindness, self-command, to stand. We reserve a sharper determination which will seek its merit not in saying anything new, but in saying that which is old more distinctly.

the further from the middle term, we might readily come to the rule always to progress in both directions simultaneously and uniformly in order to bring out against increasing depths of shadow the increasingly strong light the more shiningly. If only the way were equally open in both directions and ran on in a similar manner!

God, the real centre of all moral ideas and of their limitless efficacy, the Father of man, and the head of the world, should fill the background of memory as the oldest, the first percept, to which all recollection of the mind, returning out of the confusion of life, must invariably come at last that it may rest as in its very self in the holiday of faith. But precisely because the highest must obtain a firm position even among the earliest thoughts to which the personality growing into a human being clings, and because it is, being the highest, incapable of further heightening, there is danger lest by a continuous fastening of the mind upon that one point and so simple a point, it be merely disfigured by being drawn down to what is common, nay, to what is tedious; and surely that thought which unceasingly blames and shames the human weakness of man ought not to become tedious, or it will succumb to the first dash of boldness by which the speculative impulse undertakes to build a world of its own. Were there no other alternative, it were better to keep the idea less awake in order to have it existing unmarred at the time when man needs its support in the tempests of life. But there is a means for nurturing it, strengthening it, educating it slowly, and for securing for it a constantly increasing veneration—a means which at the same time to one theoretically acquainted with the idea must be esteemed the only means. The means is to determine it continually by contrast.

And it is exactly this which, as a matter of fact, that

other direction of the progressive presentation of the world leads to quite of itself.

It is plain from reasons whose demonstration here would carry us too far that it is the duty of instruction to guide from below upward two series, separate yet ever simultaneously progressing, toward the highest immovable point in order therein to connect them ultimately. These series may be distinguished by the names Cognition and Sympathy. Of course, the series of Cognition begins with exercises for sharpening sense-perception, and for the first elaboration of it and the nearest experience; in short, with the A B C of Sense-Perception. It would be somewhat more difficult to indicate the point of the beginning for the series of progressive Sympathy, and to justify the point given. Closer consideration soon discovers that this point can not lie in the present actuality. The sphere of children is too contracted and is traversed too soon. The sphere of adult life in the cultured man is too high and too much determined by relations which we would not make comprehensible to a little boy even if we could. But the time series of history ends in the present, and in the beginnings of culture among the Greeks a luminous point is fixed for every subsequent generation by the classical presentations of an ideal boyhood era in the Homeric poems. If one is not afraid to let the noblest among languages precede in instruction the accepted learned language, there will be avoided, on the one hand, innumerable false incidences and displacements in whatsoever pertains to insight into literature, the history of man, of opinions, of arts, etc.;* and, on the other hand, we are sure to offer to the interest of the boy events and per-

* This subject is so important and rich that it would require a book by itself. The author does not write without striking experi-

sonalities he can completely master, and whence he can make a transition to infinitely manifold reflections on humanity and society and on the dependence of both on a higher power.

The earliest education of the feelings of a child would have to be a total failure if the moral impression of those ancient tales remaining after the pleasure in the entertainment has been satisfied could be at all dubious. To begin with, the relation of fable to truth and of crudity to culture must everywhere stand out saliently for a boy who compares that picture with the circle in which he lives. And again, the two oppositions between the men of the poet and the members of his own family whom he loves and honours, and especially between those gods and the Providence whom his thought shapes after the image of his parents and worships after their example, produce in a youthful mind that has been kept pure exactly the opposite effect of that produced in people who, in the place of the tedium of lengthy discourses on religion, seek a refuge in phantasies which they dare boldly play with, and a substitute in the exercises of an art in which they have hopes of admiring their own mastership. The boy plays in the actual world; in play he realizes his phantasies. Should there be one so unhappy as to envy the Deity his nonsensuous realm and require in it a vacuum for his fictions, he must, I am convinced, have little outer life. His diet should be improved, and his gymnastic exercises increased.

However, the world as he contemplates it in his hours of seriousness ought to expand further and further.

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ence of practicability. It should be mentioned that there are many reasons for giving preference to the Odyssey over the Iliad. But at the end of the tenth year it would be too late to begin thus.

Though, to be sure, remaining always situate between the same extremes, it should, so to speak, push them to farther and farther distances, so as to make room for the multitude of characters entering along the thread of history, a light being cast on each, if possible, from his first classical describer; or, if that be not possible, at least by the reflection which from the purest sources of historic light diffuses over the darker stretches. Periods which no master has described and whose spirit no poet breathes are of little value to education. But if languages are taught for the sake of the writings in them, it is oddly done to deprive these writings of their interest by accounts which anticipate the contents in insipid extracts, especially in the silly tone pretending to imitate childhood. The continuous study of maturing youth should be bestowed on modern history. In the earlier world, especially in the ancient world, the boy may leisurely ramble if, as he should, he commenced his Homer just as soon as he had outgrown the pressing necessitousness of childhood.

"Give every one his due!" Let us give its due to this adage itself in every presentation, contemplation, and elucidation of the manifold personalities. The cleanliness and neatness shown in all genuine poetry when setting up and grouping individualities, it is the educator's first duty if not to imitate at least to receive at her hands gratefully and to make use of carefully. But the picture that he is to set up is not framed in. It is open and wide as the world. Hence the peculiarities distinctive of the species of poetry all drop away, and every weakness, every wickedness, otherwise pleading its excuse in the intent of artistic work, stands revealed in its nakedness. Conscience accompanies us to the opera, despite the vigorous protests of the poet whom the educator, on the strength of Plato's authority, banishes from his sphere, except where the

truth, the distinctness of the evil can and will serve for purifying the better, for heightening the good.

While thus through the reading of poets and historians, through the growing knowledge of men, and through moral and religious discourses which help to work up the materials previously furnished, the moral distinctions become continuously sharper, the observation of shades of character and the estimation of their distances according to ethical measure more and more correct, and by this very means the elements of the moral idea of God ever stronger in clearness and dignity, there becomes prominent on the side of cognition with increasing definiteness the concept of Nature as a system of forces and motions which, strictly continuing in the course once begun, design for us the pattern of law, and of order, and of sharply determinate measure. How defective the presentation of the universe would be, how little there would be comprised in it of the data of reality, how like a fable it would hover in the airy realm of thought, if we were to omit Nature! And how little would it answer the spirit of a rationally shaped life! Do you believe that you can by moral ideas alone teach how to act? Man stands in the midst of Nature, himself a part of it, his innermost soul traversed by currents of her power, retorting upon this external might his own might and after his own kind, according to his own essence, first thinking, then willing, then acting. Through his volition runs the chain of Nature. But for any one definite will it does so at some one definite place. This fate, originating solely from the individuality of the position, which for every determinate member of a species is necessarily a peculiar one, in contradistinction to the derivation from the highest plane of Nature, primarily designed by a universally ordering Providence for the species, this fate is the necessity which forces itself upon man. It is this

necessity which he must needs see and reflect upon in order rightly to determine his steps and the measure of his steps at every single instant. For the moral idea, though, to be sure, it speaks to our species, is silent for the individual so far as he is an individual. It knows nothing of his closest limitations. It reproves and shames, but it can not help him. It requires him at the goal. He is on the way. But it knows nothing about the way, much less can it lead him. Man must know himself and his powers, and the nearest forces to help him, and he must recognise their limitations, if their strength is to be serviceable to him according to its measure.

This fate is not the ancient *Moira*, that destroyer of life, that pure opposite of all minds.*

It can not harass the ethical man. For he does not

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* The recent reappearance of the *Moira* is a triumph for the ancient poets. Their poetical omnipotence had power so to shape the material which primeval popular belief forced upon them, that modern masters have been misled to suppose that art depends upon it. But what art, having any definite principles whatever, would count among its elements that which is entirely formless and altogether unsusceptible of formulation, as a metaphysical analysis of the concept would show? What art could suffer the introduction of an element which is completely heterogeneous to all the remaining elements and for that reason completely incapable of all purely accordant relations to them—without any intervals—which, being nothing but a collision, could only cause insoluble discords? And what cultured man has cordial sympathy for a grief resting upon a misconception rejected long since? Both, absolute fate and absolute freedom, are equally ancient remnants of crudity and equally scandalous in the domain of theory and of taste. Even if the best use be made of them in a work of art, they only help—perhaps against the will of the poet—to frame the picture by indicating the scene, the time, and point of view, and consequently the presupposed limitations, within which we may this time expect the representations of the beautiful.

demand that in him as an individual mankind or reason should perfect itself. He meets Providence, seeking to lend a hand to it in its care for the race. He hears the call to continue that which has been begun. He comprehends that theodicy is left to the action of mankind.

But what of education? How does the pupil gain the insight into these consequences of his individuality? This question beckons us to the conclusion of our essay. For man soon sees himself as Nature, provided only that he has first gained the general idea of Nature. But it can be demanded of no one to comprehend the strict conformity of Nature to law, if he has not received the strict discipline of mathematics together with its solutions.

And prior to the search for laws, there is need of the keen apperception of data. To speak generally, there is need of attention, of surrender to what lies before us. There is need of an early discipline for the roving thoughts of an early habituation to the exact continuation of commenced work. This is the sphere of the considerations to which space has been given in the introduction to the A B C of Sense-Perception.

It may be left to the thoughtful reader, if he please, to combine and to fill in these sketches. We desire to avoid incurring the appearance of having furnished a whole. But it was intended to be shown that we may still risk ignoring certain systems that can never benefit education, at least when we speak of that subject. Be that which has here been presented given over for a while to their censure. It is neither, let us hope, new nor old enough to make any one wish to fit it into foreign theories or to understand it better than the author did. In the contrary event, the author would be under the obligation to declare that he considers it a sad proof, not of strength but of feebleness of intellect, when one is inclined to shuffle together as-

sertions peculiar to different thinkers, and, above all, that he will not believe himself understood by any one to whom it still remains a riddle how determinism and morality can coexist.

Others, perhaps, dislike to find investigations so abstract in the company of an A B C of Sense-Perception. Such we beg to reflect, that after all it may, perhaps, be useful that for once a pedagogical work gives occasion for estimating the extent of the sphere of education and the magnitude of the problems still before us by the distance to be traversed in order to ascend from the lowest to the highest. And we must look out into this distance, for the last must be prepared by the first.

CHAPTER VII.

BETWEEN the appendix on the Æsthetic Presentation of the Universe and the A B C of Sense-Perception, Herbart inserted a postscript to the second edition. This postscript—a modern writer would have made it a preface—is intermediate not only locally but logically also. We learn from it that the essay we have just read was really a fragment of an older pedagogical essay, originally written to reach a pedagogical understanding with a friend.

Herbart was conscious that it would be very foolish seriously to attempt to set forth his idea of pedagogy in an appendix to an A B C of Sense-Perception. He was convinced that general ideas are bound to suffer in being submitted to public judgment without the full complement of systematic demonstration, elucidation, and illustration. If the reader has gathered a like conviction from the too brief glimpse of Herbartianism gained in perusing the preceding essay, we can only invite him to give a detailed and close reading to the General Pedagogy, the systematic work which Herbart

brought out two years later, in 1806. Being so nearly ready with the large systematic work, and realizing all the inconveniences attending the publication of the scant appendix, it may be asked why, notwithstanding, did he publish it? He was forced to do so. Herbart was not one of those who think they have changed the course of the pedagogic stars simply because they have set one of the educational landmarks. Being neither a sciolist theoretically nor an inventor of so-called devices in practice, he was especially anxious that the A B C of Sense-Perception should not be abused in either direction. He desired, by indicating the large outlines of the system of education, to make sure that the A B C of Sense-Perception would be accepted only for what it is—namely, the initial point of the knowledge series.

It was needful to cast this theoretical light upon his position on the question of sense-perception for two reasons. In the first place, the Fichtean doctrine of the Ego, as we have seen, implied a transcendental freedom of the will of which the educator can not only make nothing, but which makes education itself impossible. But the Fichtean system also implied a "productive imagination." The latter was ridiculed by Goethe in Faust, where the scholar emanating from the Idealistic school tells the devil:

> "Die Welt sie war nicht bis ich sie gebar."
> (*The world did not exist till I gave birth to it.*)

Whatever other value this Fichtean speculation might have, it as surely made an end of intellectual as transcendental freedom did of moral education. In resisting the Fichtean system Herbart was fighting for the possibility of pedagogy.

To cap the climax, Mr. Johannsen, a Fichtean, not only made a number of detailed objections to the A B C of Sense-Perception, the worth of which the reader will judge after a perusal of Herbart's reply, but attempted to make out that Herbart, the arch-realist—not (shall we repeat it?) materialist—was, after all, a Fichtean, and, what was especially galling, he clinched his pretended identification by attributing to Herbart the doctrine he himself held: "The reduction of

all knowledge to sense-perception is the highest principle of instruction."

The false light cast upon his A B C of Sense-Perception by a metaphysical system to which Herbart publicly declared himself in opposition, was the more irritating for setting out of sight his pedagogical advance upon Pestalozzianism. It has been again and again shown by pedagogic writers that that advance, to put it in psychologic language, consisted in including the Pestalozzian perception under the pedagogically far more important factor of apperception. We have seen in the former papers that Herbart very well understood the superiority of his point of view to Pestalozzi's. Still, in the first edition of his A B C of Sense-Perception he had hoped only to offer a book which might fall into line with the Pestalozzian movement. But when the Pestalozzian A B C of Sense-Perception insisted on using the unmathematical quadrangle instead of the mathematical triangle, when it emphasized relations of measurement, neglecting angular position, and when it prided itself on being closely related to objectification of number work, Herbart could not help seeing the lowness of the plane on which Pestalozzi was destined to remain. That sort of work would never get beyond sense-perception. That, by the way, is the reason why in our primary schools we find it so often wasteful of time and vacant of intellect. Herbart, accordingly, drew aside from the current discussions, and having already, as the reader will see in the work itself, given frequent hints of its dependence on pedagogical main questions, now waived the previous question of Pestalozzianism or non-Pestalozzianism altogether, preferring to take a direct appeal to the idea of pedagogy itself. This was the second reason why he was obliged to set forth that idea in the preliminary platform of principles which is furnished in the Æsthetic Presentation, the General Pedagogy being as yet unpublished. It is also the reason why we were obliged to ask a careful study for the Æsthetic Presentation.

Much clearness will be gained on the difference between the mechanique of the Pestalozzian and the Herbartian

sense-perception work by directing especial attention to the last few paragraphs of this postscript. Perception can be assisted either by adding a network or by momentarily dropping out of attention all but the most important points—the latter to be used as location centres. The former is the Pestalozzian mechanique, the latter is Herbart's plan. There is far more in that difference than meets the eye.

In conclusion, let us say that the reader will not find a few moments' reflection on Herbart's difference between analytic and synthetic instruction at all time wasted.

The Postscript to the Second Edition of the A B C of Sense-Perception.

At its first appearance the present work might hope to receive an appropriate position among the new plans of study that would be called forth and recommended by the enterprise of Pestalozzi. It might the rather seek to gain such a recommendation, it would seem, as it removed the offence which at least connoisseurs in mathematics might take at the quadrangle disfiguring the idea of Pestalozzi's A B C of Sense-Perception. It seemed advisable to show the facility of this improvement before animadversion was heard. That care may have been hasty. The critics have not objected. The Pestalozzian A B C of Sense-Perception has appeared. It has persisted in the quadrangle. It wishes, at least, to bear the title of Objective Teaching of Proportions, though it does not absolutely refuse to include anything else. Finally, it takes pride in its close relation to objectified arithmetic, with which, in fact, it coincides almost. If this be three faults for one, they should be the worse on account of each putting a fair face on the others. Although the veneration of the author for the noble Swiss is in nowise abated, he must, accordingly, somewhat withdraw his book out of

the external connection in which it stood, if only that he may not be obtrusive. The question, To what extent does it promote the plan of Pestalozzi? can no longer remain an acceptable standard for its appraisement. Being, however, too small to stand alone, there remains nothing but to connect it the more closely with the pure idea of pedagogy itself, to which, in fact, from the first it appealed in a variety of ways.

Speaking now without regard to this book, it is a general impossibility either rightly to carry out the pedagogical elaboration of any one subject of instruction or afterward to judge it rightly except there be before the minds both of the workman and the judge a similar concept of the whole of which the specialty is to be a part. Otherwise the elaborator will fall in love with the subject which he is immersed in, while the critic, who casts but scattered glances upon it, will rightly or wrongly suspect the former of that recommendatory emphasis in his proposals which is the consequence of one-sided preference merely. Against any one such preference it would, indeed, be easy to pit some other having as good, if not better, claims to interpose in education the force of its zeal. If, for example, one man praises mathematics as indispensable in education, the praise is read by another as if the indispensable wheel in the machine were pretended to be the only motor force. If the former merely uses the exercises in sense-perception so as to make them at the same time preparatory exercises for the mathematical comprehension of that which is perceived, a third man will watch him jealously in the interest of mere sense-perception, which, he says, must after all in the beginning provide everything, the concepts being found later on, perhaps of themselves. Among misunderstandings of that kind, small wonder if other objectors ask whether any one will say that sense-

perception and theoretical concepts ever change into either moral or religious feeling; whether to retain children in the sphere of sense be not obviously and flatly the contrary of directing them to the supersensuous, by which may be awakened faith, love, and hope in the tender and pure souls of children—an awakening the more needful to them the more the times are threatening to bring about the conditions by which men who have grown up coarse can only become the more hardened.

If the very idea of pedagogy could enter into the discussion, it certainly would know how to make peace between the parties by repelling none; by, on the contrary, conceding the necessity for the work of each; by then showing what and where each contributes to the whole, and how one must make preparations for the work of the other; and how exactly, because in this respect order and agreement have not been the rule, the supervisors of the entire business were forced into giving many general precepts of moderation solely that none of the workers might brush up against and hinder another; but how also, just because no one has been allowed to work with all his power, because nothing has been carried out to the end and executed largely and boldly, the whole has fallen into such a condition of feebleness as almost to draw down upon it contempt, as it can hardly be denied that hitherto the most eminent men have not on an average been exactly those who have been the most carefully educated.

Unfortunately, however, it is almost impossible to mention the pure idea of pedagogy without awakening fresh and more stirring contentions, since—where is this idea? Pestalozzi made his attempt without any system. Mr. Ith elucidated his plan by a comparison with Kantian principles. Mr. Johannsen, on page 202 of his Critique of the Pestalozzian Method, calls this the "most unfortu-

nate idea one could have." It might easily happen to Mr. Johannsen to have some one in turn say to him, There is one idea even more unfortunate, to wit, implicating the Fichtean theories with this matter, that being a scheme comparable only to passing the A B C of Pestalozzi from without into the circle of Fichte's productive intuition, with a suggestion that, by way of useful exercise, it transfer its images into the squares proffered for the sake of rectification.*

The writer is obliged here to insert some remarks against Mr. Johannsen in which the essential point is that Mr. Johannsen imagines in the present work "upon the whole" the same point of view which he himself chose for his Critique: "The reduction of all knowledge to sense-perception is the highest principle of instruction." This language is very natural in an adherent of the system of productive intuition. But it is not the language of the determined opponent of that system; and such the writer, notwithstanding his veneration for the genius of his great teacher Fichte, is forced, it seems, here publicly to confess himself, since the utterances on philosophy in the Introduction to the A B C of Sense-Perception have not sufficed to ward off that most repulsive species of officiousness, the substitution of another's opinion in the place of one's own. As for the rest, that this book is in-

* There is in the system of Fichte a call to spontaneous activity, by which is meant what we should describe as spontaneous activity in the world of sense. But the call as well as that which calls is originally the product of the imagination of him who is called. Will Mr. Johannsen undertake to indicate the point in Fichte's system at which the productive intuition itself makes everything that is external (and which, if there is to be for it a teacher and an educator, must also make him), can be touched by a call from without in any manner, however mediate?

tended, in the first place, and in the main, to be an A B C of Sense-Perception, and only secondarily and incidentally because in the nature of the case both coincide, as a prologue to mathematics—not only to higher mathematics—it is almost ridiculous to repeat since the whole arrangement of the work witnesses it. But if the work be intended to practise the measurement by the eye, why does it not measure? Why spurn the square "when in the whole of geometry it is more correct to regard the triangle as half of a quadrangle of the same base and altitude, and thus to measure by means of the square, than to make the transition from the triangle to the square?" This is correct in "the whole of" geometry in the place where it teaches how to determine the superficial content of triangles—that is to say, in a single proposition of the whole science. In this book, accordingly, this proposition occurs where it belongs, namely, in the first episode. It does not, however, belong to the main business, since surface measurement does not at all pertain to sense-perception. On the contrary, the pure concept of the content of a surface is an absolutely nonsensuous concept, destructive of all form and, consequently, of all sense-perception. Its object is to indicate purely the amount of superficial extent, disregarding altogether the fact whether the amount appear within curved boundaries or within rectilinear boundaries. From this point of view all changes to squares or cubes or other rectilinear figures are to be regarded. The object in these operations is not to straighten out that which is crooked nor to bend that which is straight, but to throw away the straight as well as the crooked altogether, so as to have nothing left over but the amount of spatial extension, to which afterward, of course, any form you please may be given. As a matter of fact, for the sake of in some way fixing for the imagination and the senses

that which has been measured, we do usually give it the form whose contours are most easily determinable from the originally assumed linear measure. It is for this reason that superficial content is represented in square, physical content in cubical form; it is not tied to the form in the least. Finally, the reminder that in teaching the sense-perception of form care should also have been taken of the apperception of curvilinear forms would be very acute if it were only possible to do more for this purpose than has been done. Comparisons of the terminal points of curved lines with reference to the radius of curvature are demanded as early as Number I of the First Section. But such comparison presupposes triangles. Drawing circles, giving separate arcs in degrees, finding the centre and consequently also the radius—all these exercises are demanded on page 186; and they are so important as to deserve to be recommended here once more emphatically. It would be possible to connect these exercises with what would be really the essential work—namely, the estimation of all, even of continuously varying curvatures at every point—by comparison with a circle of determinate radius, as an arc of which circle the determined curve could be regarded. But how is such estimation to be taught? Only empirically? That may be done, but then, it being empirical, no further instructions are necessary. Or with the same certainty and accuracy which was possible in the apperception of triangles—that is to say, by introducing the mathematical concepts? No one will entertain this idea who is acquainted with the computations for radii of curvature and for angles of curvature, together with the presupposed equations for possible curves. But further talk on the subject is inadvisable. We have to do with an author who has found "profound geometrical and trigonometrical elements" in a book that plunges into the

profundities of mathematical science no further than proportion.

Thus much in defence against the assurance of a critic attempting to introduce into pedagogy not only the Fichtean system, but also, and without possessing the originality of Fichte, the Fichtean tone. The first part of the attempt, considered as an exercise of speculation, could be none other than interesting, though it might be a question which would be the most vulnerable—the existing pedagogy, the Fichtean system, or the man who steps between the two in order to apply the system.* The second

* The system does not admit of two absolute Egos affecting each other. Who, then, is to be the Absolute? The pupil? That excludes the educator. The educator? At first it should seem that this is the only alternative even on other grounds, for I presume it is he who should speak of education. Now, will the educator, positing the pupil as a rational creature, ascribe to him also a productive intuition, among the products of which occurs, besides other things, an image of the educator together with images of his total activity, and of altogether the same world of sense in which the educator finds himself living and acting? Where is the foundation for this pre-established harmony between two originally produced worlds? Here one is almost tempted to call in the Absolute of Schelling, certainly as it, too, must dissolve into products of the Ego. But let the foundation be where and what you please, how is the educator to regard his activity? Is he to act in his world of sense by drawing and presenting figures, by speaking, admonishing, and punishing only for the sake of having a corresponding event occur by virtue of the harmony in the sense-world produced by the pupil? Is, then, the harmony, in respect to its subject-matter, pre-established so little as to await the arbitrary action of the educator? Again, why does not the sensuous knowledge, nay, the entire insight and disposition of the pupil, advance far more rapidly, seeing the two productive intuitions are so closely connected that, so it seems, there should be need only of production in the teacher of that which is to be learned and thought in order to have it actually learned and thought by the pupil. These questions will suffice as a preliminary

part of the attempt, let us hope, will not succeed. Many are offended and many will oppose it.

In these days of systems, no matter how much trouble one takes to make every thought clear by its own perspicuity, a wrong light is cast upon it from all sides, and everybody sees with dazed eyes. What is one to do but to cast a reflection upon it one's self from general principles, certain as may be one's conviction that in such a process these general principles must suffer, needing more than anything else to be kept in their rightful position, and to be exposed to public judgment only with their complete systematic environment? To set forth the general idea of education directly after an A B C of Sense-Perception in an

exercise for forward reformers of education. By way of solution, compare not only Fichte's Natural Law, but also his Ethics, pp. 280 ff., and his Destiny of Man, pp. 283 ff. Then see how far you can get with it. It is, however, easy to see why, accepting the Fichtean principles, difficulties occur in pedagogy that do not make themselves so strongly felt in jurisprudence and ethics. The reason is that in the latter cases it is necessary only to explain how we are impelled from within to posit rational beings who, according to the realistic view, are without us. The philosopher stands alone upon his transcendental point. Will anybody follow him? Will anybody understand him? He expects it, perhaps. He does not know it. He can not communicate the intellectual intuition to anybody. But in pedagogy he is forced to declare whether the rational beings posited by him are so posited as to be able themselves to become occupants of his transcendental point. For it would have to be the purpose of education to elevate them to that point, since otherwise, as a common business, education is obliged to creep in the realistic depths. More than this. He is compelled to a definite declaration as to the conditions under which one reaches the point of partly pure theory, partly pure ethics. If to this he again replies by the word freedom, education is done for; only, we must take care that he does not restrict his freedom by all sorts of conditions, but limits it in fact to the pure faculty of making throughout an absolute beginning.

appendix is an attempt that can not be intended very seriously, or it would be a great, a blameworthy folly. But a fragment of an older essay, originally written to arrive at an understanding with a friend, may well be used by the contrast of its large though merely indicated contours to make the A B C of Sense-Perception appear the small point in the expanse of the educational sphere which it really is—a point noticeable for this reason only, that it fixes the beginning of a line running far on and into manifold branches. Perhaps it will be an incidental advantage if on this occasion the possibility is shown for philosophizing on education, even independently of the most recent systems.

To return, however, as far as this postscript is concerned, to the A B C of Sense-Perception itself, whence we started this discussion. The confusion resulting from two different executions of one and the same idea exists and can not now be helped. As no contention either will or should be had on it, nothing remains for persons interested in the subject but orientation by their own reflection. By the following view the author seeks to put himself into the course of such a reflection, natural, entirely ingenuous, and not calculated for any predetermined result.

We do not ordinarily see the objects that hover before our eyes as sharply as we wish. If we desire to sharpen our vision by an auxiliary more potent than merely looking repeatedly, there are offered for our choice two ways: we may add something to what we see, or we may omit something. We might add to it certain lines of more regular, more apperceivable shape; in this case we should apperceive the object to the extent that it conforms to the lines—catching it, so to speak, in a net. But this kind of net would not allow itself to be untangled again; it would remain hung in our imagination. We should no

longer be struck immediately by the peculiarity of the object. The inclusiveness of its shape would be gone; as grown fast to foreign forms it would make one whole with them, as a part with other parts. Its hovering freely would be gone, since it would stand shored up and pressed in as though by a scaffolding, or in a cage. The æsthetical perception would be sacrificed and the impressive perception would be accustomed to a wretched mechanism. This will be the result, let the lines which we add be triangles, quadrangles, circles, or what you will.

The second way, that of omitting something from too complicated a subject, is still open. We, similarly, omit something from too extensive a historical subject—that is to say, not an entire half, not any essential portion, but lesser details, sometimes, with beginners, even the shadings of characterization and the entire evolution of events, in order primarily to impress upon the memory surely and accurately only a few of the chief chronological moments and their distances apart. Similarly, again, from a scientific presentation, which was not comprehended at the first continuous lecture, one for facilitation singles out the technical terms, defines the separate concepts to which they pertain, and goes on to note which of these concepts are comprehended in the principles, which in the demonstrations, which in the result. In a similar way, in objects of sense-perception, one might consider what we can best dispense with in recognising the shape. In viewing the object one might determine upon ignoring, so to speak, the most dispensable elements, upon withdrawing them from the attention. Should the remainder still be too complicated, we might again disregard certain elements, keeping others within sight. We should thus continue, till at length only a few prominent portions or contours should remain. But to come to the simplest element, we

should be exclusively restricted to certain prominent points. These would seem to have an accidental, unordered position after effacing, so to speak, all that mediated between them. Nevertheless, this unordered position would be none other than the true one which they had in the object itself. The apperception of this position would be compatible, therefore, with the apperception of the object. Nay, it would belong to the same; it could not be dispensed with either in impressive perception or in æsthetic perception. Furthermore, it would furnish the foundation for both, the whole shape now seeming fixed to these points. Nevertheless, this fastening imports nothing heterogeneous and disfiguring. The object would lean on nothing but itself, and stand as freely as before. But it would stand. It would no longer hover before the eye, which vacillates itself.

But this would not remove all the difficulties. The position of scattered points was to be apperceived, and to be apperceived immediately, just as those points happened to be located, without any further auxiliaries. For nothing, as our reflection above showed, must be mixed up with them. But is such an apperception easy? Make trial with the stars in the sky.

Now we are coming to the triangles. For in order to reduce this problem again to its simplest elements, one would be obliged once more to omit some of the points. How many? So many as to leave a remnant barely sufficient for mutual position. In taking, for the purpose, four of them instead of three, one would not be far astray. But it is a pity that these four points scattered at haphazard would be likely to form a very odd, oblique-angled, and obliquely located quadrangle. A square, a rectangle, would be out of expectation unless the draughtsman inserted it. Endeavouring to apperceive this oblique triangle, one

would probably look in turn at all its corners; take together each corner with two other points, and therefore immediately hit upon the triangles into which the quadrangle must divide the moment that one draws diagonals. The problem would reduce, then, to whether one knows how to apperceive these triangles. Until these simple thoughts are agreed to it would be useless to send after the book what has been held back. The author, however, can not here help noting briefly from his General Pedagogy that he deems necessary an analytic and a synthetic line in instruction simultaneous, except that the former should be a few steps ahead. Obviously the A B C of Sense-Perception belongs to synthetic instruction.

Now let our line of vision, from the A B C of Sense-Perception to the general problem of education, be reversed. Let the A B C fall back into distance. It has not been seen rightly as long as it has only been viewed near by. We seek the heights of pedagogy. But no one is invited who does not know that which is good; no one who thinks he knows something higher. To him who denies the principles, we in turn deny the truth of his.

PART II.

THE A B C of Sense-Perception is the only work in which Herbart has elaborated a given branch of instruction, the General Pedagogy and the Outlines of Pedagogic Lectures treating, as the titles imply, of the whole science of pedagogy as it is conceived under the Herbartian system. The A B C of Sense-Perception is therefore the only work which the teacher can take immediately into the schoolroom and apply it. After the insight acquired by the perusal of Part I into the principles of Herbartianism there should be no difficulty.

As it is essential that the reader should demonstrate to himself his own proficiency by handling Herbartian work, we place before him a translation of the A B C of Sense-Perception, without note or comment.

PESTALOZZI'S IDEA OF AN A B C OF SENSE-PERCEPTION INVESTIGATED AND SCIENTIFICALLY CARRIED OUT AS A CYCLE OF PRELIMINARY EXERCISE IN THE APPERCEPTION OF FORMS.

INTRODUCTION.

I. Sense-Perception is capable of Cultivation.

THE A B C of Sense-Perception rests upon two presuppositions—first, that seeing is an art; second, that the apprentice in this as in every other art must go through a certain series of exercises.

We do not all see everything alike. The same horizon offers much to one eye and little to another. To one man

it reveals the beautiful, to another the useful, to still another it is a map which he has learned by heart. In one and the same landscape a boy seeks the steeples, castles, villages, persons known to him, always clinging to single points, while a painter groups the different portions, and a geometer compares the heights of the mountains. A child is pleased with the bright and variegated. The Chinese invented the most beautiful colors, the Greeks the most beautiful forms. The amateur in silhouettes, snatching the profile of the passer-by, cuts it out with the scissors. The painter of refined taste often feels a portrait to be unsuccessful, despite hours of sitting and frequently improved sketches. Some people are born draughtsmen, and able to represent any formation of their fancy. Others are incapable of reproducing anything—at the first motion of the pencil the most beautiful apparition seems cancelled. There are occasional moments of inspiration; a painting, a view, becomes of a sudden brilliant, transfigured; only now does everything seem to belong together, to nestle together, to gain proportion, to fall into its proper positions of breadth, height, and depth; to expand and to become unified. What is it that having been concealed so long becomes now for the first time visible? What causes one man to see beauty and another to have exactness of visual perception? What is it that makes one person cling to colour while revealing to another the forms? What is it that helps one man while disturbing another, when both endeavour to reproduce a scene or a thought?

Perhaps these differences may be partly explained by the accident of special interests or peculiarities of temperament, which habituate the attention in one way or another. But we do not now inquire into remote causes. The immediate cause consists, doubtless, in the difference

in the perception itself. How and in what respect is it possible for vision to change while the object remains identical? The discovery of exercises for cultivating sense-perception depends on this point.

Strictly speaking, what the vision perceives in objects is colour. That the object is a solid body, that it is hard or soft, dry or damp, is matter of feeling, not of sight. But colour occupies a place. There are boundaries at which it terminates, or portions where it shades off into other colours. Where an object ceases to show colour it ceases to show itself. At such points are the boundaries of its visible appearance. These boundaries inclose its figure. Sight, then, in addition to colour, shows figure or form, but the latter only by means of the former. Without colour the figure would be empty; it would be nothing.

If any point in an object is of striking colour or brilliancy the impression on the eye from that point is stronger. At the expense of the unity of apperception, the vision is enticed to that one point. The remainder of the object escapes our perception, if not wholly, at all events partially. The impression made is weak, indistinct, and wavering, unless equilibrium of vision is restored by an especial attention to the whole.

Once the vision becomes accustomed to yield to glitter, brilliancy, and variegation, it is lost to a great part of the objects of Nature, and it is wholly lost to taste. In order to confer charms upon anything it will attach to it something bright, no matter if by doing so its form is distorted. Man, the most beautiful shape in Nature, is without glitter. In order to be worth looking at he decks himself in gold and purple, the feathers of birds, and opalescent shells. Colour at the expense of form is a characteristic of all tasteless ornamentation. It is easy to call to mind illus-

trative instances of savage and semicivilized nations, of our own past history, and of much that is not yet past. Clearly, therefore, the cardinal fault of uneducated sight consists in adherence to colour. More exactly speaking, it consists in being immersed in the pre-eminent colour, in losing weaker colours at the instance of the stronger. Correctness of sense-perception, which is opposed to this fault, consists in synthetically connecting everything that pertains to the form of a thing. It is attention to form to which our vision requires to be especially educated. This being gained, the sensations of contrast between light and shadow, and between shades of colour, will come of themselves almost. It will be said, perhaps, that we should mention proportion also; but the proportions are implied in every form actually coming before our eyes. We shall speak later on of the analysis of actual forms into pure form and measure.

It seems easy, then, to indicate the direction in which we must turn the exertions of our A B C of Sense-Perception. But it is quite a question how anything can be practically taught and learned that escapes almost all expression through descriptive language. Not only the eye of the artist but even the eye of the curious child is wonderfully mobile and pliant. There are quick and manifold changes from grasp to penetration, and *vice versa.* How are we to follow by words this enjoyable perusal of forms, now gliding along wave lines, now gazing fixedly at the whole, and again grouping large, small, and the smallest portions, while receiving simultaneously both the truth and the beauty of forms? And, again, what an amount of analysis, of denudation, when the elevated, receding, rounded body is made to surrender its naked outline to the flat canvas! And what a rebirth when the object of vision, originally a whole, is reproduced by single lines

and dots! The imagination had to dissolve it completely and to recompose it completely, without the falsification of a single trait. To every operation of the hand there is a corresponding operation of mind. When these operations do not proceed of themselves, what is the use of talking about them? Can the artist do more than show how? Can the apprentice do more than try? Instruction meets defeat so often in the competition with delight and genius!

Hence it is becoming that instruction should, abstaining from arrogance, offer simply that which it has to offer. This treatise does not relate to æsthetic perception. It is confined to the common sense-perception which aims to perceive a given object accurately and to preserve it faithfully. The idea was presented by the genius of Pestalozzi. Our reasons for deviating from him in the execution will be developed in Section First. If, especially, The Ground Plan of a Theory of Sense-perception should find attentive readers, it will be unnecessary to discuss minutely the alternative value of square and triangle. Pestalozzi calls his work "The Object Teaching of Proportion." But for this neither square nor triangle, but the simple straight line, should serve as the foundation. Besides, these exercises in the perception of mere measure would turn out to be so simple, to offer so little connected occupation, as to be suitable for forming youthful games rather than a branch of instruction. Aside from this, it will appear further on that such exercises are intermingled unavoidably with instruction in the apperception of forms. If the latter be given, those exercises need no specific care.

Whatever the doubts entertained as to the possibility of cultivating sense-perception by teaching, the fact itself of its susceptibility to cultivation is sufficiently clear simply

from its changeableness, from its transition from crudeness to artistic perfection.

II. The Pedagogical Value of Trained Sense-Perception.

Sense-perception is the most important among the educative occupations of childhood and boyhood.

The more quietly, the more deliberately, the less playfully the child observes things, the more solid the foundations it is laying for its future knowledge and judgment. The child is divided between desiring, observing, and fancying. Which of the three should we wish to have the preponderance? Neither the first nor the third; out of desiring and fancying originates the controlling power of whims and delusion. From observation, on the other hand, originates a knowledge of the nature of things. Such knowledge produces submission to recognised necessity, the only compulsion Rousseau approved and recommended, and which in its turn originates well-advised activity and a thoughtful choice of means.

In fancying and play, it is true, the child makes the first beginning in the working-up of its apperception material by furnishing to itself opportunities, on the one hand, for still further observations; on the other, for discovering the relations and connections of what it has observed. But to the extent that fancy follows these relations and connections, both in the sense of tracing them and of yielding to them, to the extent that any guidance whatever is accepted by it from the nature of things, fancy is making a transition to thought and to æsthetic perception; it finds the true and the beautiful. Pure fancying, mere mixing of reminiscences, taking no notice of the resultant absurdities, is nothing but a crude manifestation of spiritual existence, crude life, material whose quantity may be very desirable, but whose excellence and

worth depend on a quality which it is still to receive. Praising a man's imaginative power is much like calling one happy because he is rich.

We do not throw away riches, neither should we despotically pluck the wings of fancy, nor poison its atmosphere — namely, natural, healthy cheerfulness — by unnecessary coercion and repression. But fantasy needs guidance, and the desires need a counterpoise. Both are procured by keener attention to the realities of things. In the case of children this means, in the first place, a keener seeing of things as they actually present themselves.

No instruction is adapted to boyhood but instruction through sense-perception. In our day, fortunately, one can not enter into detailed argument on so well known a point without becoming tedious. But sense-perception instructs in no other way than by actual, definite, undistracted, keenly apperceiving vision. Accurate observation of the differences of shape is the only security against confusion and substitution. So it is in natural history, in topography, and in every kind of imagination dependent on vision, needed by the artist and the artisan in order to represent to himself the component parts of an implement, a machine, or an edifice.

In order, however, to draw every advantage that is possible from a cultivated sense-perception we should, from the first, train not the eye only, but the other senses as well, the ear especially; and further on, as a continuation to sensuous practice, we ought to cultivate every species of observation. This falls to general pedagogy. The A B C of Sense-Perception can only enter into such applications as will supplement and convert into practical dexterity its own teachings. These applications will be discussed in the last section.

III. The Cultivation of Sense-Perception falls within the Sphere of Mathematics.

The contemplation of an object which is placed before us proceeds, to be sure, of itself without scientific aid; but when curiosity flags, sight becomes negligent and dull. Just at this point the embarrassment of the educator begins. From what direction can he get a hold upon vision? How is he to set about directing it anew and persistently to the object until sense-perception has attained its complete maturity? That commands, incitements, and calls for "attention!"—the teacher's habitual and momentary expedients—can never produce pure and genuine attention is self-evident.

How to arouse and hold the attention is an important preliminary problem to all education. A little further on we shall have occasion to say something more general about it. The present inquiry relates to sense-perception only—to wit, to the apperception of form—which, as we have remarked, is to prevent immersion in individual colours.

Let us consider, in the first place, the difference between the crude and the mature sense-perception, in order to find what hope there is of changing the one into the other.

Crude sense-perception happens involuntarily on the presentation of the object to an open eye. The mind can not then help seeing, being to that extent subject to Nature. This perception is perfect from the first. To a sound eye, the object at the first glance appears in the only way in which it can appear in its present light and in its present position. Unfavourable conditions as to light and position are not the fault of the sense-perception. Neither is crude sense-perception improved by turning

and twisting the object. We are speaking of improving the apperception, considered as an operation of the mind, in properly appropriating what is offered to it. As a man with his eyes open sees necessarily that which he sees, he would as necessarily retain the received picture unaltered in his memory if it were not for an influx of other impressions.

But is it possible even to look around without perceiving a complex mass of most diverse forms? Is it possible for a child that has been carried out for an hour's airing to keep in any degree separate and unconfused the manifold sights it met with? What is similar coalesces, what is dissimilar conflicts; it cancels. The chaos that is left over gathers and accumulates from day to day, from year to year. Every novelty presented to us falls, in the first place, into this chaos. Whatever the memory desires to preserve clear and definite must be withdrawn from this chaos by a prolonged act of attention. Sense-perception without attention is crude, then, not because it represents the object incorrectly in the moment of vision, but because it leaves only a wavering, dissolving image, no longer discriminate from other images of similar objects. By way of illustration, think of a dog, in general, without determining the species. Your presentation will attain no concreteness, for a definite image would belong to some definite species. It is true that actual sense-perception is seldom crude to the degree of leaving an impression so very fluctuating and formless. We probably remember whether an actual dog we met was a greyhound or setter; but nevertheless, if one be unable to identify the particular greyhound among a number of other greyhounds, the sense-image of the greyhound has been apperceived imperfectly. It has sustained injury within the imagination; it has lost its definiteness, its distinctive marks, its individuality.

To mature sense-perception this ought not to happen. It should have been prevented by a prolonged attention, rendering the first act of vision so vigorous as to make it impossible for the image to become disfigured. Attentive vision does not, in fact, cease till it has made sure of the image. Look at an animal, a person, or, better still, a map—for in the case of the latter the difficulties become more tangible by reason of the irregularity of the outline. Now turn away your glance, attempt to represent to yourself what you have seen. Look again—you will find the renewed sense-perception correcting the already distorted picture of the imagination. If you repeat this a few times, further sense-perception at length refuses to correct the picture; the perception is now mature. From this process the keen vision of excited curiosity differs by nothing but greater rapidity. It is not interrupted. It does not relinquish the object. The testing of the image and the renewal of sense-perception coincide. The moment during which the image would be distorted, were sight to relinquish its object prior to the maturity of intuition, can not gain duration.

What is seen is confirmed, therefore, in the mind without art, provided the involuntary effect of the desire for an exact vision is long enough and strong enough. But the mere decision or general determination to become acquainted with an object is not as successful as the involuntary desire. He who does not see in the very instant that which he wills to see, sees as a rule but half, even with the best of intentions. If the object is large, if the forms are complicated, if the bulk of the gain did not occur in the first pleasurable excitement, exertion strains in vain. The shapes become but the more confused, the more distorted. A favourable moment must be awaited; only, the fitting time is not apt to last long.

And how much worse all this becomes in instruction! Even if one could always count upon the good intentions of the boy, the power of voluntary determination—i. e., the power of good intention—is incomparably weaker in the boy than in the man. Besides, instruction is obliged to advance simultaneously along different lines. Supposing a good plan of study, there is always manifestly great detriment, when the different lines of progress do not coincide at the right time. But to complete the case: If the teacher is to guide onward simultaneously a whole school, must not we abandon the attempt of getting a hold upon the mind through the senses; the attempt at producing through the presentations of natural objects, instruments, plastic works, and pictures, a system of mature sense-perceptions and images, upon which to erect, as upon an essential part of its foundation, the edifice of instruction?

What if we had it in our power to make a beginning by controlling the sense-perceptions through the mind? The thought may seem paradoxical. And yet we all know that the eye is nothing without the discipline of the mind; that only through the latter have we learned to estimate distances; that the babe, though seeing an object, does not know how to grasp it; that we translate unceasingly the perspective appearances of things into their actual shapes. Perception finds easily, if the mind seeks understandingly. We grasp differences sharply and spontaneously, when we know beforehand what distinction is to be made.

Supposing the possibility of first bringing the mind to the point of taking exact note of all the possible differences in forms, it is entirely probable that the eye would afterward be sufficiently attentive to perceive the differences where they exist. If we could only gain a boy's patience for that single thing, his curiosity would do the rest

as soon as it was even moderately interested in the objects presented.

The question, therefore, is, How can forms, exclusively as forms, be studied systematically? It is an almost equivalent question to ask, How can we teach sense-perception? For what we do systematically, we do in accordance with concepts. And it is concepts alone that can be put with accuracy into words; that can be formulated into definite precepts, and can as such be transmitted from the teacher to the pupil.

All that the greatest minds of all the ages have done toward the apperception of form through concepts, we find gathered into a single great science—mathematics. Above all, then, unless pedagogy is to risk exhausting itself in unavailing endeavours, it must first, for the indicated purpose, search among the treasures of mathematics.

But pedagogy and mathematics are things often so widely separate in practice that we may the rather presume, perhaps, upon the reader's indulgence in detaining him upon a few preliminary observations before attempting the problem itself.

IV. On the Pedagogical Use of Mathematics.

The conceptual field from which education anticipated aid in solving the present problem is the best cultivated among all the regions of human knowledge; doubtless, because it is the most susceptible of culture. No other species of knowledge can so readily be raised to the rank of scientific demonstration as that which determines form by number, because these concepts are the most comprehensible of all. Mathematical science, therefore, is not only the most complete and the readiest to furnish aid, but its help is also pre-eminently welcome as being most closely related to the nature of human thought.

In fact, there exists in every mind, which, without a thorough knowledge of arithmetic and geometry, has familiarized itself with ideas and branches of knowledge that in their nature are later productions of human thought, a disproportion of development whose magnitude can be estimated most easily by a glance into history, showing how many preparatory stages had to be traversed in order to reach the one and how many to reach the other class of ideas.

Though these observations are of some weight, there are reasons of far greater urgency to recommend to pedagogy the use of mathematics. Reflection upon the following, though but briefly indicated, considerations may enable the reader to judge whether it would be an exaggeration to call mathematics indispensable to the beginning, the middle, and the end of such instruction as the duties of education require of us.

As to the beginning. In the first place, let us glance back at the preceding. It has been shown that we must through concepts gain control of the vision which is to fix the forms and which depends but imperfectly upon intentions and reflection, and for which definite description and direct communication and instruction are still less possible. These concepts, being concepts of magnitude, will belong to mathematics. Now, the educator will not, of course, disdain the aid which they perhaps may offer. Yet he will prefer to have his pupils become so disposed that sense-perception be brought at once by the first involuntary exactness and tenacity of attention surely and strongly to a maturity such as needs no further aids; and as the ultimate result of the exercises which we shall propose, and which only at first will be somewhat in the nature of aids, we intend to produce exactly such a predisposition in the case of sense-perception. But does the

demand for a predisposition to attention apply only to sense-perception? The educator needs it everywhere and always. Let him seek to obtain it universally. It will benefit sense-perception among other things. The fact is, sense-perception does not require attention by any means as much nor as absolutely and necessarily as do all matters of feeling—history, morals, religion—all that concerns mankind. These are subjects where attention admits no adventitious aid. Here we lose not time only, not enjoyment only, but the marrow of education itself, if the freshness of first presentation ages without impressiveness; if tasteless repetitions spin out tediously what should have seized upon the interest quickly; the very maxims and forms of expression in which the fulness of conviction loves to express and condense itself, are—misused and soulless—buried like corpses in the tombs of memory.

Do you hope simply by the manner of presentation, by your personal conduct, to gain for these subjects that swift, toilless attention which is the mother of feeling? Much more will be obtained by preparations reaching back into the past. But a universal negative condition of success, in this as in every pedagogical endeavour, is that the pupil never permit himself to be inattentive when the teacher speaks.

Inattention, however, is the natural condition of the learning boy. Were you not teaching him, his flow of presentations would not on that account rest. His play or, if he were denied that, his fancies would occupy him with all the vivacity of youthful spirit. They are repressed by instruction. Instruction, however, suffers in its turn from their importunity.

In order to obtain power over them let it be the first care of instruction, not less than of the instructor's personality, to gain the respect of the pupil. Let the instruc-

tion proclaim itself, not in words, of course, but in deeds, as an absolute sovereignty of the understanding, by which one is hurried along inevitably, it being impossible to refuse a single step. Just as the educator must procure prompt obedience for every one of his express commands, so the instruction itself can not afford to have any of its assertions misunderstood, or to leave unnoticed or forgotten even its smallest qualifications. If such mistakes occur, and at first they will happen every moment, they must surely and wholly betray themselves. On the other hand, they must lie bare to inquiry; it must be impossible to cover them up, to cloak them, or to minimize them. Even the magnitude of the mistake must be undeniable, must manifest itself in measure and number. Inwardly, on the other hand, the mistakes must make themselves felt by the consciousness of striking embarrassment. Complete incomprehensibility must at once darken suddenly the brightness of light; everything must fail; no device must succeed so long as the mistake remains. Everything must at once return to the secure and even tenor of its way as soon as the fault has been removed. Every self-deception, pretending to understand what it does not understand, to be conversant with that with which it is not conversant, must thus come to light. The weakness of his logic must be plainly evident to the pupil; but not only his weakness, also his strength, and his capacity for development such an instruction must show him. It must lead him to demonstrate them to himself by his deeds. That which seemed incomprehensible, unattainable, that before which his mental powers stood still, must become perfectly clear, and clearness must lead to perfect ease of execution.

All these traits will be recognised at once as belonging only to mathematics. Yet we may be permitted first to review the remaining subjects of instruction, so as to see

whether the other branches are able, each as its own fruitage, to generate the authority which they need, to say the least, as much, or whether they require to have this fruitage grown on the stem of mathematics, for transference to them.

The subjects appealing to the feelings are too tender, too easily injured, and of too high a dignity to be obliged to undergo the rude task of combating boyish inattention. Let them peacefully dwell nearest the heart. Like feminine beauties, they should be carefully fostered. Their charms require attention—charms which must not wither.

Linguistic studies, geography, and natural history are matters of memory. They must be made the subjects of a variety of quizzes and reviews. They bear the appearance, therefore, of being well suited for the training of the attention. Unfortunately, however, these subjects as a rule become interesting and useful only to one equipped with a good memory for them, who, experiencing nothing of the painfulness of retention, enjoys the ease with which he runs through the complex variety, as one enjoys a distant and variegated view. But to one to whom this view lies plunged in mist, who slowly recalls the details, anxiously counting piece by piece that he may lose nothing, the multiplicity of names becomes the more repugnant the greater the number of quizzes and reviews. It is true the names are gradually fixed in the memory, but the fixation is not a sensible gain. Knowledge does not by this means grow. It does not push on. It does not reach out round about it. It unriddles nothing. It does not enhance the fulness of thought as is done by mathematical insight, which increases even by a mere dwelling upon the problem.

Chemistry alone, perhaps, does approach somewhat more nearly to this insight. The science of chemistry in

its present state is probably as yet altogether too recent for its pedagogical powers to have been noted sufficiently. It deals with a number of combinations whose transmutations, presupposing a knowledge of affinities, the student can find by his own reflection. Partly in this respect and partly in respect to the consequences derivable from experiments, chemistry offers to the pupil riches by self-occupation—a charm which is heightened by the uncertainty, by the semi-darkness in which light in many places is rather dawn than day. When one has a pupil on whom this charm seizes, and for whom it is needful, such a science, happily bringing to its disciple both reward and labour, should be warmly welcomed. Because it rather easily admits of test questions, and because it reveals any lack of attention by many wrong results, it may at times, for the purpose that we are considering, be even preferable to mathematics. Nevertheless, there is this inconvenience, in schools especially, that one has to show too many materials, and that one can not venture to leave them to the uncontrolled spontaneity of children, but is obliged to withdraw them almost altogether from their hands. In the worst case, when the teacher is restricted to merely relating all the experiments, this science becomes completely unserviceable. To conclude, chemistry is not a science for children, because it does not lie within the horizon of common sensuous observation. It presupposes a way of looking at nature which is broadened by a variety of information. Therefore what we have said above as to its being preferable under certain circumstances to mathematics applies only, but in this case with full force, to youths whose attention, from lack of earlier correct guidance, has as yet attained no steadiness, and hence still requires separate measures for making it firm and robust.

But form and number are at the very centre of our

original mental horizon. The elements of measuring and reckoning are the most natural—the first, the almost unavoidable pre-exercises, which even the weakest understanding procures for itself spontaneously. Subsequent mathematical elaboration connects most closely with these elements, and gradually proceeds from them by unbroken sequence. On concepts of magnitude, above all, the teacher is able to attain himself and to require of the pupil perfect verbal expression. In mathematics nothing eludes language, nothing need be avoided in the details of discussion. There are no emotions to spare, there is no tedium to fear, unless the subject be treated beneath its dignity. Here, then, in the place where we find the means of cultivating sense-perception, we must seek also what is to be found nowhere else—namely, the guiding thread for an early instruction of children that shall be of such a quality as to procure, both for itself and all other teaching, an authority at whose behest inattention shall flee and attention come and persist.

The considerations so far presented upon the importance of mathematics for the beginnings of education are strictly appropriate here. They relate immediately to the A B C of Sense-Perception—not, it is true, as to its purpose, the cultivation of sense-perception, but as to the material which belongs to mathematics. The subsequent remarks, upon the other hand, on the indispensability of mathematical science for the middle and the end of education, deserve a place here only to the extent that they give occasion for casting a glance from the A B C of Sense-Perception at the whole field of education. For it certainly is necessary that in considering even the smallest portion the educator should have in mind the whole. If he pursue any one idea singly, the gain oozes away because it is not taken up, and the other measures of instruction

are unprofitably restricted and confused. The idea of the plan must be in perfectly equal balance. That is the only salvation for the works of an art which is more than others exposed to the most untoward accidents. But thus also there is hope of controlling the accidents, since fate acts by single impacts which often cancel each other, while art commands a system of forces which pursue the same purpose through a considerable series of years.

As to the middle portion of education, everything might be repeated that has ever been said concerning the usefulness of mathematics for the cultivation of the mind. Being a gymnastics of the thinking powers, needful even in the earlier years of childhood, shall we be able to dispense with it later on? The mind as well as the body must from time to time return to its gymnasia in order to test its muscles and to renew their perfect elasticity. Add to this, the influence of mathematics on the remaining sciences. Without it, what becomes of physics, of knowledge in the arts and of machinery? But these reasons, though they be long since recognised, have little effect upon pedagogists. That very field of instruction toward which the noble science is directed with the greatest effect, interests them the least, namely, the investigation of nature; and for purposes of instruction they employ any other information concerning nature rather than the mathematical. Let those versed in the matter repeat to them ever so often that without this means of education everything drops apart into miserable fragments, they still confide in these fragments as having a power of serving in an incomprehensibly useful manner for youthful—let us say—diversions. The cardinal fault seems to be that we have not yet assigned to the investigation of nature its true place and rank among the forces that must co-operate in the mind of an educated person, and hence in a mind

that is being educated. Whenever this is done, Mathematics, the indispensable handmaid, will also soon be put in possession of her rights; and an external mark of her actual possession will be that we shall no longer await the last years of instruction in order to scatter a few forlorn samples of the long ignored mathematical science, samples which, being deprived of all introduction and connection, nauseate the student with tedium, and hence are inevitably surrendered by him to swift oblivion; but that there will be assigned, on the contrary, to geometry and lower algebra such a place in the middle of instruction as will articulate them with proper preparatory exercises and enable them to extend a real influence through all the subsequent instruction of youth.

As regards the finishing time of education, among several reflections here, too, urgently calling for the aid of mathematics, there arises a principal one, the essence of which may be briefly indicated by saying: The true perfecter of education is philosophy; but the office of mathematics is to obviate the dangers of philosophy.

It is in the nature of Philosophy to isolate general concepts and to remove them for a time out of the sphere of their actual applicability. It is its first and indispensable office to separate and purify the concept which she singles out for investigation from the adventitious qualities and modes connected with it in the mass of data out of which she sets it forth. The concept thus stripped gains clearness and definiteness at the simultaneous expense of the disappearance of the limits within which, and of the conditions upon which, it had reality. True, this lack of limitation is, in fact, an absence of any idea of magnitude. Thus, apart from its conditions, the concept ought to be viewed merely as a subject of thought; being and nonbeing are not to be predicated of it.

But by an extremely frequent confusion there is substituted, for lack of limitation, infinitude, or else universality and perfection; making of this disregard of conditions either unconditioned reality—that is, absolute being—or logical impossibility; the latter when the contradictions reveal themselves which must inevitably originate in a concept that is torn out of its necessary connection. Occasionally we meet with the really ridiculous case of the last two mistakes, although cancelling each other, being committed by one and the same thinker, who, while acknowledging contradictions inherent in his concept, nevertheless ascribes to the concept unconditioned reality.

It is really these inherent contradictions which ought to be motives for the progress of reasoning, for when determined sharply enough they are bound to teach us how to discover the complement which the concept lost when abstracted from actuality, or the series of complements, if several of them existed. The investigation being absolutely complete, the contradictions will be removed as completely, because, in connection with the complements the concept did have reality, and to ascribe inherent contradictions to a reality would be the acme of self-contradictory nonsense and the destruction of all thinking. Such philosophical integrations would be to the well-known mathematical integration as genus to species. It is true that in mathematics we never hear about those inherent contradictions, but they can be shown immediately in any differential, if we will ignore for a moment what the mathematician never forgets, namely, that the differential is necessarily connected with its integral.

It is self-evident that the procedure which we have indicated would fulfil the main purpose of all theoretical philosophy. For the necessary connection of all data would become manifest by our finding upon investigation

that the isolation of concepts is impossible, that each requires the other in order to fuse with it into a single whole. As yet, unfortunately, it is the mathematical species of integration almost alone which prevents the whole genus from being an empty name. That species already flourishes and thrives excellently. It is the highest glory of speculation. Hence, also, it is naturally the sole prototype for any future labours in philosophy, and the sole preliminary exercise for youth, even if we intend to do no more than make clear to them the defects in philosophical attempts which have hitherto been made.

However detrimental the mistakes likely to attach themselves to the failure to notice conditions become to theoretical philosophy, the forgetfulness of limits is beneficial in practical philosophy. The heart, oppressed by the limitations, expands the unfettered concept into a genuinely Platonic idea. The concept becomes infinite; it becomes perfect. To think perfection is the happiness of the spirit and the origin of the better life. Truth, beauty, goodness—these ideas are born so. What is actual, Plato says, desires to resemble them, but it can not. But it is this that a noble enthusiasm does not quietly suffer; it is this that impels and that ought to impel it to help the actual by strenuous exertions to approach the ideal.

It is precisely at this lofty elevation that the great dangers of philosophy begin. Does the impulse to act need nothing but the idea of the good in order actually to attain the good? Does zeal need no bridle? Does the weight need no pendulum, preserving rhythm and measure?

These questions our age answers and comments upon plainly enough. But it speaks less plainly concerning the remedy of the evils, concerning the complement which education, while quickening and firing youth through

philosophy, must needs articulate with the latter that it may not spur him beyond all bounds.

This complement can be found nowhere but in the field of ideas. The mind elated with ideals despises all external constraint. It defies, hurls back the threat with its entire strength. What holds back the mind must ruin it in order to be secure from it. This type of spirit the educator recognises even in the boy, and when he meets it he rejoices greatly; for out of this species of wildness is formed by education the most beautiful, the most willing, the most faithful obedience. It tames itself as soon as we teach it that it should, not that it must, do so, and how it can do so.

Do not destroy the noble zeal which causes you anxiety, but habituate the youth to look upon the things of this world as only gradually capable of conformation to the good. Habituate him to look upon them as magnitudes and upon their changes as functions of the motor forces, or, in other words, as necessary results of efficient causes, absolutely accordant to law, despite all apparent irregularity, and perfectly definite in every step of their progress. Show him that wherever knowledge has penetrated the phantom of irregularity has vanished, and that knowledge has progressed successfully wherever it has sought for magnitude and measure. Lay bare to him the ridiculous conceit of ignorance inclined in the past and inclined to-day to deny the existence of law unless it be manifestly obvious. Reveal to him the wonders of analysis. Teach him how the uniform progress of the abscissa controls surely and accurately all the bendings, turns, and nodes of the manifold curves. Show him how heedfully the swift hyperbola shoots along its asymptote so as never to touch though eternally approaching it. Show him how even the infinitely small angle of curvature which eludes

ll number, all measure, can not escape comparative com-utation and determination. Teach him how to compre-end this marvel. Let him seek and find for himself ow all these concepts of magnitude connect together and e produced by one another. Let him discover them in ature, and so let him perceive that all these curious curves rve but as symbols for the host of movements and changes hich in the actual world go on before his eyes. Thus will learn how to observe. Even where law is invisible him, he will seek it nevertheless; at least presuppose Against this law, whether known or unknown, he will ely abstain from flying into passionate fury. He will lize that in the actual world the question is not what wishes, but what, by very different rules, follows from act. He will cautiously accommodate himself to these les. The rules themselves he will endeavour to intro-ce into the service of that preconceived idea of the od, and to retain them in it. He will thus view man o as nature, and as a nature amenable to culture, de-spite all chimeras of a radical good and evil. In all this fear nothing of materialism! The scholar knows his general concepts of magnitude too thoroughly to forget that in spect to them the concept of matter is as accidental a one as that of mind. Moreover, daily experience, which he never loses sight of, will guard him sufficiently against commingling unlawfully such different applications of the same theory.

Thus much for matters that seem so very distant from the A B C of Sense-Perception, though, to be sure, they are not; for, to enable us to attain these higher purposes, the A B C is necessary. The use of the integral calculus by the youth will never amount to anything unless the boy was well grounded in his elementary exercises.

It remains to add something in respect to pedagogical

economy. Financial objections destroy the finest plans. I the pedagogist, time is the costly possession to be carefu husbanded in distribution to the various occupations t present claims to it. Suppose that we have proved indispensability of mathematics to a complete educat as irrefragably as any mathematical theorem itself. S that which is immediately necessary in life, in busir vocations, enforces claims both older and more imp tive. Moral culture by its very nature is under the es cial protection of the educator. In its aid we can avoid introducing some things æsthetically beautiful, to mention the smuggling routes by which the comn ties of mental luxury spontaneously introduce themse Finally, and above all, we can not escape providing that information whose absence would betray a vu ignorance. When all this, well analyzed into its divis and subdivisions, is, as it were, set forth in a tabulat side by side, in order to separate the most necessary fr the less needful and to assign to each its year and ho the pedagogist can not help being frightened at the fe ful amount. Nor can he help pitying himself and e poor brain of his pupil into which so many and s ch heterogeneous things require to be crowded. The p s-pect becomes altogether gloomy when we recollect that, after all, what is termed science originated in the first place in a real and invaluable intellectual pleasure of the discoverer, whence to cheer and elevate the mind rem ns the true purpose of science; and now when all t ese benefits are about to impress hourly the brain of the boy, not only will the brain be oppressed by them, but the heart also, the deeper, delicate, sympathetic sensibility, will be strained, pulled, and dragged by them in the most opposite directions. For, necessarily, pleasure in any one study must cause its opposite in so many others which

interfere as disturbances. With the bolder mind, unwilling to submit to this division of his feelings, education must be at continual war. On the beautiful and gentle soul, which will not pardon to itself any lack of docility, education will inflict an uninterrupted series of injuries. Instead of helping the rising ideas, education will destroy them one by another. Instead of enlivening the feelings with a warmth ever new, it will chill and kill them one by another.

If the author were to mention the starting point of a pedagogical insight that reaches to the foundations of education, he would say it is profound reflection on this truth. It is such a reflection by which Pestalozzi is urged to seek for definite series in instruction, and to such a reflection we owe his A B C of Sense-Perception.

He who really knows the mathematics of which we are speaking, he who has not only learned it, but who has tasted, as it were, the feeling it is capable of affording, can not possibly counsel that it be forced additionally upon youths that have already accommodated themselves to the usual course of instruction, with such interest as is possible for the latter. The state of the feelings in mathematical thinking differs too widely from the mental tone of one seeking with youthful hope philosophic wisdom, or lovingly investigating ancient history, or giving himself up to the songs of the poets. These states of feeling are not changeable from hour to hour, like clothes. Students, whose interest has become fractionally excited for all these and for several other branches, would feel only still more confused by the priestess of definiteness and clearness—Mathematics. They would be completely at a loss which side to turn to. Perfectly passive obedience to their teachers would be their sole refuge.

But this whole way of thinking on the subjects of in-

struction as of a mass with mutually extraneous parts, a way of thinking followed by pedagogical theorists—not, to be sure, systematically, but very commonly—is radically wrong. The opposition is somewhat like that in physics between the atomistic and dynamic systems. According to the latter, it is far from true that the whole quantity of matter in space is mutually excluding. So, likewise, the pupil's mind should not give up as many separate energies, as many little pieces of his total capacity to learn as acts of apperception are required in instruction. On the contrary, his capacity of learning is an intensive magnitude to be constantly occupied upon a corresponding solidity of instruction in a continual effort. We can not here do what is really needful—namely, explicate this with speculative precision. But this much will be seen easily: First, that there is no more advantageous way of escaping from the embarrassment caused by the lack of time and multifariousness of instruction than by increasing and strengthening the intrinsic content and weightiness of that which is taught every hour, for by this means a great many of the divisions and subdivisions before made will disappear by condensation; second, that every hour of solid instruction bequeaths energy to the pupil's mind, and that one ought to conserve the energies generated by different species of instruction, and to this end to guard them from feeling and acting against each other; otherwise there will be caused that conflict of emotions and that intellectual stupefaction which exclude all independence of character; third, that, on the contrary, we ought to use energies that have been generated unitedly and to the greatest possible advantage, in order thus to win more and more energy; fourth, that, as a consequence, in distributing instruction by years and hours we should above all consider which of these forces are the most serviceable and the

strongest, in order, in the first place and with the greatest care, to procure these. The whole course should be so arranged as never to let any energy lie fallow, but, on the contrary, so as to cause all previously generated energies to act in all succeeding time with their full force at every stage.

If this be the calculation, mathematics is sure to find room enough in the programme of studies.

We shall grudge to it neither its constituting, at three different periods, a principal branch of instruction, nor its being kept present to the mind and increasing in familiarity during the intervals by interpolated exercises.

Not counting the earliest exercises in numeration, measuring, etc., mathematics will make its first appearance at the eighth, ninth, or tenth year, in the form of the A B C of Sense-Perception, and require a period of about three fourths of a year, one lesson daily and a few hours' practice. At the twelfth, the thirteenth, or the fourteenth year, a period of a year and a half, and again one lesson daily ought to be sufficient to make arithmetic, geometry, trigonometry, and lower algebra perfectly plain to a student who has been prepared by the A B C. Finally, at the end of the eighteenth, the nineteenth, or the twentieth year, higher analysis, again allowing a period of a year and a half and a lesson daily, would complete the study as far as any but a professional mathematician would care to master it, as a means to further culture and for use in the course of life. This approximate estimate considers pure mathematics only. As far as pedagogy is concerned, applied mathematics always pertains to the realm of its subject-matter.

But the lessons must not be given less frequently than once a day if we wish to count upon anything like the

necessary mental concentration being maintained by the student.

As for the exercises to be interspersed in the intervals, the other branches of instruction will offer a multiplicity of opportunities.

V. *Some Observations on the Exposition of Mathematics for Educative Purposes.*

If it intends actually to furnish the previously mentioned advantages to the education of youth, mathematics will fall into line with the other co-operative agents as sociably, as amicably, as possible. It certainly will bring with it all its real dignity, and it will show it all. But it will fain avoid all peculiarities that are accidental. As far as possible it will speak and act like the others. Where it seeks to improve their conduct it will guide it back to Nature. But it will not seek to bring in new fashions, or to put new restraints and stiff mannerism in the place of the old. Assuredly it will never renounce the preciseness of its peculiar diction. But the higher the rank which is granted to it on that score, the more carefully it will watch that its diction shall ever be real precision. It will take heed lest it be proved that mathematics by its technical language lulled itself into thoughtlessness, and performed its work asleep and mechanically. It would still be great Homer sleeping; but Homer would cease to be a model for those reverently listening for the sound of his voice.

To speak more definitely: In order to furnish preparatory exercise in thinking, mathematical reasoning must not be a peculiar kind of thinking, but it must follow the course naturally and universally taken by a sound understanding which is not hindered in its reflections by accidental disturbances.

Now, it is the way of a sound understanding first to look about it from the point whence it desires to progress, so as to survey the whole field for the purpose of orientation. In the next place, it is its habit to take the shortest way to its goal and to remain always conscious of its bearings. Finally, having reached the goal, it is wont to look about once more in order to familiarize itself with the new neighbourhood which now surrounds it.

Conformably to the use of language, we may with especial propriety ascribe the survey to the imagination and the progressing to the understanding.

It is obvious at once how thoroughly applicable the distinction is to mathematics. That great science gives to the imagination at least as much occupation as to the faculty for drawing conclusions. Before the latter can commence to demonstrate, the former must have sketched the figures, it must have pierced the bodies by manifold lines and intersected them by planes, it must have stretched out the infinite series, it must have interwoven them with other series. All the plenitude of combinatory presentations pertains to the imagination. The merely progressive understanding would creep with sad slowness from element to element. It is precisely the general view of the series and of the different values of a variable magnitude that is most difficult to a beginner in analysis. The pupils have conquered roots and logarithms as soon as they can present to their minds with ease the accelerated increase of the powers simultaneously with the uniform increments of the roots or exponents, and the increasing compactness and simplicity of the logarithms simultaneously with the uniform growth of the roots.

What is the result when, without preparing and accustoming the pupil to a general survey, there is used in a

computation, say, an isolated root or an isolated logarithm, or even, if you like, a few of them, but of whose necessary ratio to each other the student forms no conception? Do not you feel how timidly, how anxiously your pupil is compelled to proceed on the narrow rope of the rule, his eyes clinging to his feet exclusively? And what becomes of the case when we heap up a multitude of general theorems concerning things so unfamiliar? We are compelled, in order to give some measure of assistance, to waste time by numerous examples which, nevertheless, being far too isolated in the wide sphere of the concept, bring but little gain to insight.

On the contrary, make it your first law of presentation not to neglect the mathematical imagination. Early habituate the imagination swiftly to run along the entire continuous series contained under a general concept—a rule of great influence even in very different kinds of instruction. It follows from this that we should from the very beginning, as far as possible, so teach as to cause all magnitudes to be considered as fluctional. This will excite a desire for the whole of mathematics.

Neither is it a matter of indifference what material is first presented to the imagination. It must be easy to work up, and its working up must be of the greatest possible advantage to all future study. There is nothing that, so to speak, seems to lie so nearly at the centre of mathematics as trigonometry. The consideration of triangles is fundamental to all geometry; and pure analysis, which, strictly speaking, has nothing in common with concepts of space, would frequently not know where to turn in integrating if it did not borrow sequences of proportions from trigonometry. This consideration alone would make it desirable, provided always that we found it compatible with the other purposes of the A B C of Sense-

Perception, that triangles should become the first subject for mathematical exercises.

As concerns, in the second place, the relation of mathematics to the understanding, may the great science pardon its admirer if he do not, in this respect, find it as perfect for the purpose of mental culture, its noblest use, as it must indeed become. It is not that mathematics lacks extent, or certainty, or conciseness. But it does lack systematic elegance and philosophical perspicuity. Every defect of the sort becomes most disagreeably noticeable, most disadvantageously important in pedagogical use; since for this use not the results are important nor their reliability, but the thinking, and its exemplariness of procedure.

Close speculative thinking admits no arbitrariness. It should contain neither more nor less than just so much as is needful to complete an immediate insight into the inner necessity of the theorem under discussion. All acts of arbitrariness are idiosyncrasies on the part of the inventors and instructors. They both delay universal communication and are unworthy of it.

Mathematical analysis every moment allows itself liberties in the way of conveniences in which no precise method can possibly indulge. To prove a theorem by analysis, by resolution of the concepts, means to let one's self be impelled by the given concepts themselves to the concepts that contain the inner necessity of the proposition. But this necessity does not lie in arbitrary auxiliary lines or in arbitrary computations. In fact, it is not discovered at all as long as there are two or several proofs making the matter equally plain. It is certain that that to which the given concepts impel, that which they can require, is only that which necessarily and essentially pertains to the nature of the theorem. But, for this very

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itself as a duty of education. But our introduction was needed to show the bond connecting sense-perception and education with mathematics. As between sense-perception and mathematics, this, so far, is a very loose bond. Only the very general conclusion, namely, What is done according to plan is done according to concepts, pointed, from the necessity of systematically caring for the completeness of sense-perceptions, to the science which elaborates the concepts of intuition. If there was to be a teaching of sense-perception, it was clear that this, like all real teaching, would have to consist in a transmission of concepts. Still it remained obscure whether and to what extent it would be possible to place the plasticity of sense-perception under the control of instruction. How could it help remaining obscure, as long as we had not more profoundly investigated the nature of intuition? But it was not the business of the introduction to do so. It will be the most essential element in the investigations which we are now about to enter upon.

This investigation will, then, in the first place, determine the material for our preliminary exercises. To bring this material under concepts, and to convert the latter into theorems, will, in the second place, be our requirement of mathematics. We expect a preliminary reply containing a general promise; some information as to the way in which we may expect to be aided by mathematics; what it will give; what it will withhold; and how much regard it will have to itself in doing so. To conclude, pedagogy will indicate what external form it wishes for the whole, what conveniences it can proffer, and, on the other hand, what facilities it expects to be proffered to itself. Mathematics thus steps in between the sense-perception and education. The following considerations are arranged accordingly:

I. Outlines of a Theory of Sense-Perception.

This theory has nothing to do with external vision, its organ, light, etc. It is perfectly silent, therefore, on the subjects of perspective and optics. Just as little does it enter into the subject of æsthetic perception. It may, therefore, be very brief; for it only remains to explicate the act of sense-perception, the immediate mental perception and fixation of what is visible. Only, be it borne in mind that the sense-perception here discussed is the perception of form—in other words, the synthesis of what is coloured. (See Introduction, first number.) We do not in the least discuss the question through what "window" things-in-themselves step into the soul.

That which the eye sees is never simple. It has always an extension of breadth and length, though not of thickness. We here merely recall these well-known propositions. Now, upon a surface which in its extension is uniformly visible to the eye, the merely physical eye, considered by itself, would receive uniform impression, and for that reason discriminate no forms; for a form is limited. In order to be seen it must be lifted forth out of that surface by a special act of attention. But there are some things in this plane more prominent than others, which is to say that one thing is perceived with greater or less strength than another. This inequality of perception may and really does vary. Sometimes there is more that is strongly perceived, sometimes less. Sometimes there is a tendency to concentrate vision on certain points. Again, the eye rambles; and then, again, it gathers up a small portion of what it has contemplated in detail, then a larger, a still larger, and finally the whole. It is, without doubt, by such a synthesis of all the parts of a thing, and omission of all other things that are simultaneously visible,

that the shape of a thing is found. But if, in addition, the relative positions of different things are to be found, the several syntheses which have been made and which can not be dissolved again, must be embraced in one new comprehension. Units already composite must combine in still greater units. The process may continue indefinitely. The pupil of the eye, the eyebrows, the eyelashes, etc., taken together, constitute the eye; ears, nose, mouth, etc., together form the face; the face with the remaining members make the body; and several persons together make a group. When we look at this group with a trained eye, we do not dwell uniformly upon the whole and ignore distinctions and limits. That would produce a chaos of colours, but not a well-articulated group of people. On the contrary, one synthesis lies for us within another, in the way that we have described. While looking, we form the eye separately, the nose separately, every person separately, finally we shape them all into a single group.

Perhaps the reader is astonished at so complicated an activity, of which we are usually so little conscious. But astonishment will diminish upon remembering how imperfectly, how bunglingly the operation is often performed. The artist, to be sure, accomplishes the articulation of forms completely. But the average beholder misses both beginning and end. He attains neither to that which is smallest nor to that which is greatest; wanderingly and indefinitely his vision hovers about midway, doubts as to division, doubts as to how it is to unite the divided. Because he is struck with the demands which the object makes upon him, he imagines that it has pleased. But only the artist who masters his object enjoys it really. In some other instances, however, an unpractised eye yields itself to the pleasure of gliding up and down the gentle curvatures, thus playing over the shape and really in this

way enjoying the charm of the beautiful; æsthetic perception might, perhaps, better begin in the gliding vision rather than the effort to fix the apperception. Only it should not let that suffice. But the mistake of the untrained beholder is revealed at once when he sits down to reduce his vision to a drawing. If he attempts what is natural, namely, reproducing as he has perceived; if he attempts to have his pencil glide as gently and as gradually as the eye did, it is impossible for him to avoid falling into considerable error at the first curve whose movement he desires to reproduce; for at every point a curved line changes its course but infinitely little. Consequently, by proceeding from point to point infinitely small mistakes accrue in an infinite number, and thus slipping in unnoticed put the whole out of position. As a matter of fact, the gliding vision is not an apprehension of form. To a figure all its parts belong at once; therefore all claim a uniform notice. The unpractised draughtsman intended to let the terminal point of the curved line become the final result of all its windings; but instead of merely connecting the point of beginning with the terminal point mediately by a line leading from one to the other, he should have apprehended their distance and their position with reference to the bend of the curvature at once and immediately. Thus, his curved line would have lain between them in a fixed position.

From what has been said, it is clear that the articulation of forms is very complicated, and therefore very difficult work. To make it easy and accessible to all, it must be analyzed into its simplest constituents, that the details may be mastered first and recombined subsequently.

Speaking technically, we analyze a combination, a large synthesis into lesser syntheses, and each of these into the least syntheses. This operation is the reverse of that

shown in the theory of combinations when it progresses by degrees from given and entirely simple elements to all the more and more composite combinations that can be made out of them.

We need, therefore, here to insert the indispensable essentials of the general doctrine of nonrepetitive combination. Further instruction may be drawn from other books, as, for instance, Stahl's Elements of the Doctrine of Combinations, Jena and Leipsic, 1800, pp. 72 ff.

The given elements, whether actual things or numbers, or, as in the present case, coloured points, are customarily designated by letters. In order to represent the combination in question in a short example, let a series be given of not more than five elements. Let them be lettered a, b, c, d, e. Of these there are combined first two, then three, then four, then all five.* All combinations thus possible are shown by the following table:

a	b	c	d	e
	ab	ac	ad	ae
		bc	bd	be
			cd	ce
				de
		abc	abd	abe
			acd	ace
				ade
			bcd	bce
				bde
				cde
			abcd	abce
				abde
				acde
				bcde
				abcde

* In order to apprehend quite easily this and the next following portion, upon which everything depends, look at an engraving, a

Consider the table carefully, in order to see whether you would know how to set forth similarly all the combinations if only the four elements a, b, c, d were given. Dropping e, we drop all combinations in which it occurs. But they are all placed vertically in the last column. It is therefore necessary only to omit this column. If to the four letters a, b, c, d the letter e be once more annexed add this column again. It is as easily apparent how the table would change if a sixth element, f, were added. Another column would have to be annexed, beginning above with f running through the paired class by putting af, bf, cf, df, ef vertically under each other; through the triple combination class by having abf, acf, etc.; and concluding at the extreme lower end by a newly added sixth class, containing for the present nothing but abcdef. It is plain that in the same manner a seventh element, g, would bring in a seventh column, an eighth element an eighth column; that, in a word, all the possible combinations of no matter how large a number of things can be found on the plan indicated.

As stated above, sense-perception, intending to appropriate correctly the form of an object, must uniformly apperceive all the portions of the object—i. e., all the smallest places which might be termed apperception points. But there are innumerable points, and we intended facili-

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ground plan, or some such thing, upon which the objects, or, to speak exactly, the terminal points of the objects, are designated by letters indicating names written below. Where our table combines a couple of letters, take in the engraving the two designated points. But they can be combined only by giving attention to their relative position, or by drawing in thought the straight line possible between them. Similarly, where the following table combines three letters, take the three points indicated by them; they are found to inclose a definite triangle; four points a quadrangle, etc.

tating the task of apperception by analyzing it into its simplest constituent portions. There is, then, to precede the uniform apperception of all the points, the synthesis of a few points, in order to connect with them gradually several others. If we take the first points very close together, joining to them again the closely approximate points, and so on, till we have slowly gone from one end of the object to the other, the result will be the gliding, flowing vision, the disadvantages of which we have shown. On the contrary, then, the points first to be combined should be chosen as distant as possible, so as to fill up the intervening space step by step. It is in about this way that a draughtsman habitually proceeds, who first sketches the outermost contours, then indicates this thing and that scattered about in the middle space, and puts off till the last completing the connected filling in. Thus is assured to the picture correctness of position. But only the practised draughtsman secures it. The attempts of the beginner in sketching outlines are very uncertain, very toilsome, often tedious, and sometimes fruitless. The causes are readily apprehended. For him, the outline is both too rich and too poor: too poor, because he has had no practice in analyzing the total view of the original, so as by stripping it of its charming features to leave only the pure contours; too rich, because the contours consist of lines—lines which always contain points innumerable, and still furnish an infinitely large instead of a simple synthesis. Besides, there is persistence in the inclination to see and to draw the lines of the contour free hand, and the faults thus originating are fatiguing to his patience. Before making any drawing, sense-perception should have been cultured and confirmed on the original, not on the scant, unattractive outline which is being sketched. In the original, sense-perception should have

searched out the truly simple and main constituents of the form. After imagination has completely mastered the shape of the whole by the aid of some constituent forms, it ought to fix within every smaller whole the still smaller units which are contained in each in a manner similar to that which was at first applied to the large unit. Not till now has the time come for recombining what was singled out. Let the bodies originate from the members, the groups from the bodies. Not till last of all is it time for the proof by pencil and chalk that imagination's grasp is firm enough and sense-perception complete.

What are the simple, main constituents of form that are to be sought out in the original? An examination of our table of combinations will indicate them. Let us begin at the beginning. Unrelated points indicated by the single letters a, b, c, d are nothing for the purposes either of form or of measure. When related in pairs, as ab, ac, etc., they have between them a definite direction, a distance, a straight line. This, to be sure, is something in the way of measurement, for a line contains a certain number of inches, feet, etc. But the form of all lines is the same, be they long or short. Or rather, even here there is as yet no form. Form is produced for the first time, and hence in the simplest manner, by the combination of three points. When four or more are combined, previous combinations by threes are included. The latter may be therefore regarded as the fundamental constituent parts of all the more composite forms.

But as that which differs in measure and magnitude does not by that difference alone give us any form, let the reflection enter at once that, on the other hand, mere form determines no magnitude, as form remains the same whether it appears larger or smaller. A good por-

trait, even a miniature painting, has the same shape as its original. This distinction we shall find important later on.

In order, then, to bring to maturity the apperception of a painting, let us say, we should take together in the first place, in the main contour, three simple points as distant as possible, and prominent as terminal points of the figure a, b, c, and then join with ab in the place of c a fourth point, d ; next, acd ; and in the last place, bcd, so as to exhaust all the triple combinations of the first four points—all this for the sake of having the exactest notice taken of the mutual positions of the three points in every possible combination. In the next place we should add a fifth point, and later a sixth. Of the combinations possible with the first four points some, though for brevity not all, ought to be appereeived. We should scarcely need in addition to these a seventh or an eighth point. We should proceed only as far as suffices to fix the position of the whole main contour thoroughly in the imagination. In the case of some students this requires more, in the case of others less. In the next place, we should proceed to the contours of the parts, doing with them as we have done with the general outlines. The work should continue into the parts of parts to any extent that may seem advisable. Finally, in order to connect the smaller contours with the comprising larger contours, and all the subordinate contours with the general outline, it would only be necessary to combine the respective points into triangles in a similar manner. During this operation the table of combinations would serve for the prevention of confusion. By its means we keep our bearings among the multitudes of possibilities open to choice. By aid of the same table we might go on to fourfold, fivefold, manifold combinations. We should thus gradually seek to

guide the eye back again from the details to the uniform apperception of the whole.

But in order that we may be able thus to proceed, the presupposition must be made of an eye possessing ready skill in accurately apperceiving all the triangles—that is, every simple fundamental form—and of clearly discriminating them. For without such skill, obtained by previous practice, the many triangles originating in the process would not fail to run into fresh confusion among themselves. Unless there be previously acquired facility in the discrimination of triangular forms, the many analyses of one simple view would only serve to render the eye timid and anxious. Besides, if the triangles which occur in sense-perception be not also simultaneously thought under concepts, the teacher can not converse with the pupil concerning the object under observation. By escaping expression in language, the accuracy or inaccuracy with which the pupil has seen the triangles would be inaccessible to inquiry.

Hence we must premise to the above-described process a series of preliminary exercises, rendering facile at one and the same time the apperception and the concepts of all triangular forms. Such is, if we may be permitted the expression, the deduction of our A B C of Sense-Perception. It must be understood completely and connectedly if the reader is to be enabled to enter into the meaning of the present work.

II. On the Mathematical Determination of Elementary Forms.

If the proof just given that the true elements of all form are triangles needed further confirmation, mathematics would bear witness to it by the procedure it has in-

variably observed. It strives to master all forms by the triangles existing or possible in them.

These triangles mathematics is in the habit of rendering perceptible to the senses by actual straight lines between the terminal points. However, it is clear that these lines merely express the distances between the points; that what constitutes the triangle really is the relative position of the terminal points; that a somewhat practised eye can dispense with the sensuous objectification; and consequently that a picture or an outline which is to be apperceived by the eye would be quite wrongfully disfigured by actually drawing the triangles to be taken into contemplation. But, on the contrary, it is equally clear that objectification is all the more needed in the preliminary exercises. Here the lines inclosing the triangle must be rendered absolutely evident to the eye.

But the imagination is, by these preliminary exercises, to become familiar not merely with one or a few but with all possible triangles. It is to find an old acquaintance in any position of three points that by any possibility can be presented to the eye. Still, is not this requirement infinite? In the expanse of space, can not we scatter about three points with unlimited arbitrariness in such a multiplicity of differences as immediately to put to shame any one who should pretend to know all possible positions?

He who in this argument should seriously appeal to the infinite expanse of space must have forgotten that size has nothing to do with form—an observation which reduces very materially our belief in the manifoldness of possible triangular forms.

They may as well be represented small as large. If one side of the triangle be a foot, then this may remain the same although the shape of the triangle is altered.

On the contrary, if this side were to increase at the same ratio as the other sides, we should obtain, to be sure, other and still other magnitudes, but not any new shapes. Precisely that the shape may change, and precisely inasmuch as it does so, it is necessary for one side to remain constant as the others increase or decrease. This excludes all triangles that are mere enlargements or reductions of one another. As far as form in them is concerned there is only one triangle; on the other hand, the imagination must be practised to identify this one for any size.

Nevertheless, the multitude of possible triangular forms remains infinite, but only in the sense that between two triangles already approximate there may always be constructed infinitely many, infinitely proximate ones, making a continuous transition from the one to the other. The infinitely approximate triangles, to be sure, are no longer discriminated by the eye. But for this very reason it is possible to establish for sense-perception a certain, not excessively large, number of model triangles, of which a pair will always offer a narrowly limited place between them for any occurring triangle.

Throughout geometry we compare triangles to see whether they are equally determined by some of the angles and sides, and hence themselves equal. It being once determined by this means that they agree as to form and magnitude, or both, no further attention is given to the question what form they do and on the basis of the determining pieces must have. Of course, the eye sees it in the drawing, but takes no notice of it, as the attention is not called to it. The real scientific enouncement of the form of a triangle does not belong to geometry; it belongs to the later science of trigonometry. But the latter, though it certainly gives to the understanding general rules on the subject, does not give any correspond-

ing image to the imagination. The objectification of the teachings of trigonometry is left, then, for our exercises. Thus we have determined more clearly where in mathematics the means for cultivating sense-perception are to be found, and also the relation in which our preliminary exercises stand to the science.

The triangle, in general, is the fundamental form for sense-perception. The right-angled triangle specifically furnishes to trigonometry the fundamental concepts for determining all the other triangles. This course must also be taken by our preliminary exercises in order that they may, as far as possible, conform to the science.

As far as possible. But the true foundation of trigonometry is higher analysis, while we are constrained to borrow our foundations from experience. We merely detect certain relations from empirical measurement, while by the science their necessity is demonstrated. We take, on the faith of imperfect inductions, certain propositions whose universality the science theoretically proves.

Severity of demonstrations is inappropriate for small boys; but all the more appropriate is the multifarious objectification of numbers, fractions, computations, constantly occasioned by the triangles. This occasion to confer greater clearness on arithmetic the teacher must utilize to his utmost possibility.

We wish especially to observe that even here we shall be able to attain the advantage wished for in the introduction, of not representing individual magnitudes only, but the whole multitude of triangles as in a series, in continuous transition. Even the meaning of the differential formulæ of trigonometry can be here anticipated sensuously.

We may hope, therefore, that by one and the same occupation we shall be able to generate the mathematical

imagination, to give a preliminary training to the understanding, and to excite an interest in the whole science.

III. Pedagogical Considerations.

After the experiments of Pestalozzi, one may trust a degree more readily that education will feel strong enough not to relegate well-founded plans with such exceeding quickness to the realms of pious wishes. Especially the objectification of trigonometrical teaching is secured against every doubt which might otherwise be entertained as to its feasibility by the excellent effect of the quadrangles, circles, and horn-leaves in the Pestalozzian school. The voluntary drawing upon slates, which in that school furnishes so happy an outlet for superfluous manual activity, must also render the earliest services to trigonometry. Horn-leaves, especially, are indispensable. They should receive the first rectangular triangles, and almost exclusively guide the boy to imitate them in drawing. Children who, like those of Pestalozzi, know how to draw a circle freehand, are fully prepared to receive the trigonometrical instruction by horn-leaves, and to fulfil its behests with sufficient accuracy.

If it be desired, however, to utilize every advantage which education can by very early preparation provide, an exercise of the attention serviceable for the present purpose also is at least conceivable even for the earliest years of childhood. It can be tried, at least, without danger.* The experiment proposed would be about as follows:

* It seems that not all readers have been kind enough to read exactly what was printed, though from the first edition everything appeared exactly as here, even to emphasizing "at least conceivable." The following passage is from No. 32 of the Göttingischen Gel. Anz. of 1804: "One of Pestalozzi's adherents forgot himself so far as to make the assertion that if only the attention of all babes

As soon as the child in the cradle shows attention to external objects, hang in a convenient place opposite the cradle a dark board : only, do not have it perfectly black, for this colour is avoided by the eye of the child ; rather have it speckled brownish. Underneath, before the board, fasten a bracket, likewise of a dark colour. Put upon it every day, not by any means variegated objects of many harshly contrasting colours, but simply things of merely bright colour and having a form agreeable and easily apprehended—something that is new daily, but with repetitions of what has preceded—an egg, an orange, a shrub having but few leaves, a well-shaped cup, a saucer, a pitcher, glasses, boxes, watches ; later on some flowers, provided you do not crowd them ; finally, if you please, a bust—a whole figure. Be careful not to be too generous with bouquets, many-coloured pictures, etc. The eye is to be exercised only moderately, receiving instruction in things that it can apprehend without confusion. In addition, however, to those objects, a few yellow nails may well find room on the board. Their metallic lustre will especially attract the eye. Three of them driven far apart suffice. The triangle which they form can be changed daily. Our elementary forms may thus become the child's earliest acquaintances.

The partly systematic, partly æsthetic laws which domi-

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from the first days of their lives should be directed to bright points in order that they might firmly grasp the shape of the triangle, a concept upon which was based all the knowledge in the world, an improvement of the human species would follow, by which must vanish also those moral evils which have produced the French Revolution." I wonder what have been the steps of growth in the myth of the [bright-headed] nails, enabling it in the brief time from 1802–1804 to attain from its first childish crudity to this magnificence of adornment !

nate all education must bear some sway even in our A B C of Sense-Perception in order to give it arrangement. If we should make it a rhapsody of separate exercises strung together, it would produce in the disciple no collected power upon which one might count. Besides, it is especially fitting for occupations relating to mathematics to give the first incitement to system in a boy's mind, to accustom him to sequent and complete thought. The numbers representing the ratios of rectangular triangles serve happily as a principle supporting all subsequent computations. Nor is it a slight intellectual conquest for a boy to be able by the aid of those numbers to traverse completely the whole wide field of possible triangular figures at a uniform and measured pace.

The A B C of Sense-Perception, we grant, is only the prologue to mathematics; and it is really mathematics which, by guiding, inciting, moving and satisfying the speculative interest, should appear under the form of a work of art. But even this little prologue should contribute its preparations toward the result. Let it be clear even by itself. Let it be well rounded. Let it appeal to the senses. Above all, however, let it point from the small to the great. It should make felt everywhere the near presence of the great science. It ought sometimes to bestow a little gift in its name. By the invisible hand of the great science let it cause a knot to be loosened now and then, or a fault to be rectified. Again, by the omniscience of mathematics let faults be brought to light so as to compel their confession by the drawings, the instruments, and the imperfectness of computations. Carelessness, and misapprehension especially, must not be allowed the slightest hope of slipping through unreproved.

It is a chief requisite of a good pedagogical plan that it be flexible enough to fit the various capacities. Where

several students are to be instructed simultaneously, there is an especial need of the art of providing freedom of motion for the quicker minds without removing them out of the common road travelled by the crowd, without allowing them to gain an advance which would break up the class. The usual method of regulating the length of lessons by the capacity of the mediocre members of the class and forcing all to conform is disadvantageous, it is plain, both to a majority and to the best students. This measure is at one and the same time too large and too small, and too small especially for those whose education would reap the richest rewards. To preserve flexibility of plan we must sharply separate what belongs to the main idea essentially and necessarily from that which constitutes merely useful applications and enlargements upon it, keep ready a sufficient number of these enlargements, and know how to direct students into them with ease. Being destined for the more capable among the students, they must lead to grades somewhat higher scientifically. These requirements—admittedly difficult—we have in our presentation of the A B C of Sense-Perception made an attempt to comply with by the Episodes which we have intercalated in various places. It is not necessary to go through them completely. The teacher may use them as seems best. The first Episode might even be divided, what is said of the circle and ellipse being inserted, say, after the fifth chapter. To be sure, the teacher will require skill in the art of instructing several pupils if he wish to lead forward some without disturbing others in their reviews and drills, and without even withdrawing from them his attention. But then this work counts throughout upon pedagogical art and dexterity. It is really intended to furnish some little occasion for perfecting and refining the art. It by no means hopes to achieve the merits of

Pestalozzi's efforts, by which even the crowd of poor school teachers is to be qualified to serve as organs of an instruction easy as well as accurately measured off.

Second Section.

EXPOSITION OF THE A B C OF SENSE-PERCEPTION.

The consideration of reasons and relations was carried to a great length in the Introduction and in the First Section, in order to be the briefer in the presentation of the plan that we base upon them. Theory is always obliged to leave something to trial and experience to alter, to fill in, and to expand. Even supposing that success in practice should not fulfil our expectation, our reasons might remain valuable; only, it would be necessary to draw still more cautious conclusions from them. But a large plan is justly ridiculed if it suffer shipwreck upon trifling difficulties. It is to be hoped, however, that the little plan we shall submit will prove sufficiently definite to make possible experiments along its lines under the supervision of educated men.

I. First Beginnings.

With slate and pencil even a five or six year old child may practise drawing straight lines and combining them in various ways. In this connection seek to master completely the course of Pestalozzi. Above all, the tiresome occupation of drawing lines one after another must not be the only entertainment. On the contrary, it must be byplay only, the child being chiefly instructed by the teacher's talk, and by repeating what is said. Mouth and hand must be put in motion simultaneously: while the eye is required to master lines, the imagination and the ear must be cared for, lest they yield themselves up to too interesting impressions. There being, unfortunately,

so many things which must be learned mechanically, they mass them together in order that multitude may make amends for their lack of content!

In this manner the drawing of lines must be practised daily for weeks. To facilitate it and to avoid the necessity of oral criticism which would disturb that other instruction, inscribe on the horn-leaves the horizontals, perpendiculars, and right and left ascending and descending oblique lines, both intersecting and parallel. It can be done with great ease and accurateness with the point of a penknife and a ruler. At each lesson have the slate pencils sharpened to a point, and have the slates cleaned with water which is absolutely pure. Thus the child without accustoming his hand to disadvantageous pressure will easily copy the pattern, distinctly apparent in the horn-leaf as a fine white line, by placing the horn-leaf upon the black slate. Just as easily, accurately, and gently will the leaf when placed on the drawn line indicate to the child where and to what extent it has missed the copy. Of course, the child will use several such horn-leaves in succession. On the first let only a single line be drawn, but let all the possible positions of this line be shown, appropriately named, and imitated on the slate. Then proceed quite gradually to more complicated combinations and intersections of several lines. Later on, have the children draw circles, which in the beginning must not be made too small. The diameter should be at least two inches. The circle may subsequently be made larger and smaller.

These exercises, always intermingled with other teaching, may possibly have to be renewed now and then during several years before perfect success is attained. Only when they have been brought to perfection can we broach our subject with absolute certainty.

II. First Determinations of Measure and Form.

In order to acquaint the eye with the customary measure, and to keep the latter constantly present, let there be marked in the wooden frame of the slate the length of a foot, divided by larger strokes into halves, by smaller into fourths, and by still smaller into inches. Let the child practise copying accurately one, two, three inches. Better still, let it practise marking them off on straight lines which it has drawn, testing them by horn-leaves upon which a few inches are marked. We will here make the general remark that throughout the remainder of the book we presuppose the constant use of horn-leaves having the requisite figures cut in.

When an object is enlarged or reduced without alteration of form, we may consider its measure as enlarged or reduced correspondingly. All numbers indicating how many times the measure or its smaller subdivisions are contained in the object remain thus absolutely unaltered. In order to accustom children to this mode of viewing an object, which is necessary for what is to follow, make them copy the foot and its subdivisions on a variety of reduced scales; sometimes just as they please; sometimes the teacher may determine that the copy is to be a half, or a third, or two thirds of the true foot, etc.

The form of a thing is determined partly by the proportions of the lengths occurring in it and partly by bends and angles. We do not need both; either of the determinations suffices by itself to determine the form in which the other determination will then occur of itself and necessarily.* From this a good deal follows for the A B C of

* In a very valuable critique this passage has been blamed as an error of haste, and it is true we need only move one side of a polygon parallel to itself to have the angles appear as remaining the

Sense-Perception. It must teach determining the form by either means. It must show in addition how from one determination results the other; the latter will constitute the principal business of all the following propositions. For the present our main object is to raise the primitive apperception of forms into distinct consciousness. The proportions of the lengths are concepts. They are sometimes concepts of such difficulty as not to be understood at all without mathematical science. But the angles are sense-perceptions. Through them we immediately perceive the form; only, if they are to determine the form with accuracy, we must discriminate them very keenly. Hence the discrimination of angles will be the exercise next following.

Let the child draw a circle. Through the centre let it draw a horizontal and a vertical line. Thus the circle is divided into fourths or quadrants. Further lines drawn through the centre should cut each quadrant into thirds, or the circle into twelfths. Finally, have the thirds of these thirds, the ninths of each quadrant, marked by little strokes on the circumference. The child, being told that the smallest parts of the circumference thus originating are usually divided into parts ten times smaller, termed degrees, will now be able to count the degrees on the quadrant by tens—10, 20, 30, to 90.

Out of the figure thus originating we now copy others which are simpler: first, the horizontal and the perpen-

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same, while, nevertheless, the proportions of the sides change. But the polygon, as the term is used technically in mathematics, is unknown to the A B C of Sense-Perception. For its purposes every distance is a line, and consequently every figure is merely by the distances between the vertices divided in every possible way into triangles. Upon this rests the meaning of the book, including the criticised passage.

dicular; but only to the point where meeting they make right angles. Next, we may perhaps have the angle of 60°, represented separately once more; then the angle of 40°, of 30°, and so on. The lines forming the angles should be drawn now greater, now smaller, and again of unequal length, so as to show that nothing but the meeting of the lines constitutes the angle.

To obviate the mistake of imagining that all degrees have the same length, we may divide, in the manner indicated, larger and smaller circles, causing the same angles to be copied out from them, which will render evident that an angle of a definite number of degrees is always the same, whether taken from a larger or a smaller circle.

Again, the child having drawn upon the slate a number of circles, large and small, as different as possible, side by side, the teacher might erase from one circle one half, from another one fourth, from a third seven twelfths, ten twelfths, five twelfths, and so on, and let the child state in degrees first how large is the remaining arc, then how large is the erasure. Next, let the child recover the centre. Finally, let it restore each circle. At a later stage arcs from different circles are connected, that the child may make acquaintance with the manifold figures thus originated.

III. Right-angled and Isosceles Triangles.

The determination of the form was to depend on the angle. Consequently, we shall change the latter uniformly, describing in so doing a series of rectangular model triangles.

Trigonometry leaves to us the choice whether we will close the angle by sines or by tangents. The sines are measured by the radius—i. e., the smaller by the greater. But it is natural to the eye to apply the smaller to the

greater, so as to measure the latter by the former. Besides, the right-angled triangles formed by sines and cosines are all included in one circle. How large a circle this would have to be, if the triangles are to be presented clearly to the senses! In the drawing there would be a variegated intersection of lines, and into the computation fractions would have to be introduced representing sections of lines so small as not to be visible to the eye.

Sense-perceptibility is, for our purposes, the highest law; hence the tangents and secants are to be preferred. Those under 45° are unnecessary for this purpose. Calling in every right-angled triangle the smallest side radius and the intermediate side tangent, the angles increase from 45°. These names, it is true, are a slight infringement on mathematical usage. But the child needs unchanging, easily applicable expressions; it would become confused if now the radius, and again the tangent, were greater. When the boy is further advanced, subsequent mathematics will, by the many new things it teaches, readily change so trifling a habit into conformity to its usage.

Two horn-leaves, into which the drawings represented in Figs. 1 and 2 must be cut with absolute accuracy, are now and for everything that follows the most indispensable instruments. Fig. 2 contains only a linear measure, a square measure, and a protractor; but Fig. 1 shows the rectangular model triangles which must be as perfectly impressed upon the eye as the numbers representing their ratios upon the understanding.*

On Fig. 1 give the boy, in the first place, as the name

* It will be advantageous to have the two figures variously enlarged on other horn-leaves in order to facilitate the very important exercises in enlarging and reducing.

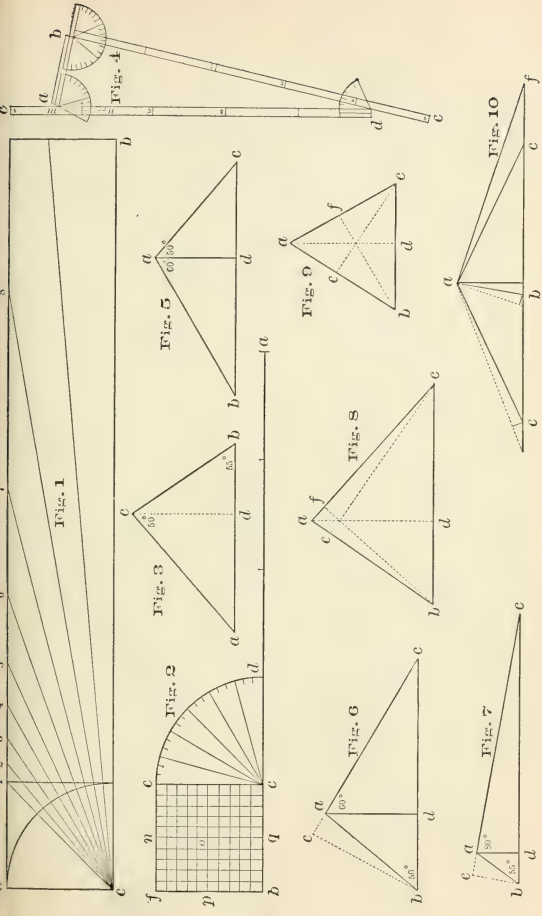
Fig. 1
Fig. 2
Fig. 3
Fig. 4
Fig. 5
Fig. 6
Fig. 7
Fig. 8
Fig. 9
Fig. 10

for the lower short horizontal *ac* the word *radius*, which designates always the distance of the centre from the circumference of a circle. Next give him *tangent* for the perpendicular, which touches the circle below at *a*, and *secant* as the term for each of the slant lines running out from the centre [hypotenuses]. In the next place, note the points at which the perpendicular is met by different secants. The length from one of these points to the nethermost end of the perpendicular where it touches the circle is strictly the tangent which belongs to each secant. Both secant and tangent depend for their size on the size of the angle formed by the secant with the horizontal radius. The smallest of these angles occurring in the drawing is one half of a right angle. It amounts, therefore, to 45°. Taking the next following secant together with the radius, the angle is 5° larger, and amounts, therefore, to 50°. The third secant with the same radius incloses an angle of 55°; and thus the angles increase each time by 5° up to 90°. The points *a*, *c*, and 1, inclose the first triangle; *ac* 2 is the second triangle; *ac* 3 the third; *ac* 4 the fourth, and so on up to *ac* 8. After this *ac* 9 ought to follow as an additional triangle. But, as appears in the figure, the ninth secant, in order to cut off the tangent, would have to be prolonged farther than there is room on the horn-leaf. Next, the ninth secant, the angle below once more progressing as before by 5°, follows the perpendicular *ab*, which therefore should furnish the secant for the angle of 90°. But when will this perpendicular intersect the tangent? It runs parallel to the latter. It hence does not approach, much less reach it. Both tangent and secant of 90°, then, continue infinitely, forming no triangle. If the angle were smaller, by however little, than 90°, the two lines would approach each other; meeting, therefore, somewhere at some time, they would inclose a

triangle. Between 1 and 2 appears the difference of the first and second tangents; for subtracting *a* 1, the first, from *a* 2, the second, the remainder is obviously the space between 1 and 2. Similarly, the difference of the second and third tangents produces the space between 2 and 3; the difference between the third and fourth, the space between 3 and 4, etc. These differences are never equal, although they originate through the angle at *c*, opening continually by equal differences. It will be seen that the larger the angle is already, before it progresses, the more will the tangent and the secant increase, though the angle enlarge but little. Imagining the angle proceeding not by 5° at once, but very slowly, very gradually, like the hand of a watch—and yet even like such a hand, not now more swiftly and again more slowly, but with absolutely uniform motion—the slightest angular progress would infallibly cause a small addition to the tangent and the secant also. But, however minute this small addition might be, nevertheless, each succeeding one must be greater than the preceding. Or thus: Tangent and secant increase at an ever-accelerating rate, when the angle increases uniformly.

Explain these considerations to the children at the start with perfect clearness. Then let them draw the first, the second, the third triangle, each separately; then all three side by side, in order that they may accustom themselves to discriminate them accurately; next the second, third, and fourth, side by side; next the third, fourth, and fifth, and so on. Do not proceed to the following three till the preceding three have been practised thoroughly. During these exercises, the instant that a triangle happens to be completed, let the child take the horn-leaf, Fig. 2, which should be drawn on precisely the same scale with Fig. 1; and by means of the five-part

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line, *ab*, have it measure the tangent and secant of the triangle it has drawn. Observe that the radius in Fig. 1 is exactly equal to one of the five parts of line *ab*. One such part let us call a unit. The scale in Fig. 2 must next be applied so as to indicate, by means of line *bc*, which is divided into ten smaller parts, how many units and tenths the tangents and secants contain. As the unit, it may be best to take the statutory inch, cutting the size of the figures on the horn-leaves in proportion. The numbers found, each child may write on the blackboard, placing a period after the number of units, and the number of tenths after the period, writing—for example, a unit and two tenths, 1·2.

Keep the child practising the drawing of the triangles till it has at least once succeeded in making each of them exactly right, and has discovered the numbers for the tangents and secants. Thus it will at length get up the following table :

For	45°	the tangent	is		1·0 ;	the secant	over	1·4.
"	50°	"	"	" almost	1·2 ;	"	"	1·5.
"	55°	"	"	" over	1·4 ;	"	"	1·7.
"	60°	"	"	" "	1·7 ;	"	exactly	2·0.
"	65°	"	"		2·1 ;	"	over	2·3.
"	70°	"	"		2·7 ;	"	"	2·9.
"	75°	"	"		3·7 ;	"	"	3·8.
"	80°	"	"		5·6 ;	"	"	5·7.
"	85°	"	"		11·4 ;	"	"	11·4.
"	90°	"	"		infinite ;	"	infinite.	

For 85° we can not avoid the teacher's giving the numbers, as, on our little figures, they can not be measured.

If the numbers so laboriously found have not impressed themselves spontaneously upon the memory, the impression must be completed by memorizing them. In

order to accustom the eye to recognise the triangles in all positions, and in order to furnish greater variety, do not during the drawing place the horn-leaf always straight, but turn it now thus, now otherwise, directing such or such a triangle to be imitated in the slanting position in which it is shown.

A few addenda will show in completion the twofold definition of the triangular model forms by the angles as well as by the proportions of the lengths.

Each triangle was completely defined, and was distinguished from all the other triangles solely by the angle at the centre of the circle, or, no circle being drawn, by the angle opposite the tangent. But in addition to this, and to the right angle common to all the triangles, we have a third angle between tangent and secant. We have it without more ado, for it is apparent that it can not be enlarged or reduced without altering the angle at the centre also. Therefore, this one being determined, so is that one. It is seen, furthermore, that as one increases the other decreases. What will be each time the size of the smaller one? To find this we have here no other resource than measurement. This purpose is served by the graduated quadrant in Fig. 2. Measurement shows that in the first triangle the angle at 1 amounts to 45°; in the second, at 2, to 40°; in the third, at 3, to 35°, and so on: in a word, we find the proposition that, taken together, the two acute angles in a right-angled triangle always make 90°. This proposition, which in geometry is demonstrated, must be retained in the memory. Either of the two angles resulting invariably from the other, by the simple subtraction of one of them from 90°, the shape of a right-angled triangle depends on either singly, not on both taken together. To be convinced by ocular demonstration, it suffices to draw the tangent and secant, joining

them at any angle, for instance, at the angle of 35°; by joining the radius at right angles to the tangent, the angle of 55° will originate of itself between radius and secant, and the triangle will show exactly the same form as if we had first joined the radius and secant at 55°, and then closed the angle rectangularly by the tangent.

It having been made plain to the child by this means how the shape is determined by either of the angles, there remains to show that the numbers previously found—namely, the proportions of lengths—also determine the shape of the triangle, no matter what its size may be. To this purpose the exercises in enlargement and reduction are serviceable. In the first place, let the child draw for himself the standard line *ab* in Fig. 2, magnifying it at pleasure. Of the arbitrary measure so originated let him take, by the guidance of the ratios in his self-made table, the appropriate amounts of units and tenths for the tangent and the secants of each triangle. In this operation the child will find himself compelled to join these two lines at exactly the same angle as when using the original measure, if his radius, being the enlarged unit, is to close his triangle without being too long or too short. Repeat this species of drawings on changing and arbitrary standards till it is thoroughly understood by the child that his measure determines nothing but magnitude, while the numbers expressing the ratios decide the form, and therefore determine also the angles of the triangle. Despite all diligence, some slight difference of shape between the original and the enlarged triangles will, nevertheless, become noticeable occasionally. These are occasions for reminding the pupil that the numbers in the table leave almost everywhere undetermined small remainders, which, though always less than a tenth, and hence of no great, are yet of some, influence upon the form. By seeking one of the numbers

once more, as at first, by measurement, and by giving close attention to the linear remainder over and above the already known number of tenths, estimating it with all possible accuracy as a half, even a quarter of a tenth, the drawing on the enlarged scale can be correspondingly rectified and brought nearer to the form indicated in the horn-leaf. Thus a desire will spring up for the more accurate determination of the numbers that the following work will gratify to some extent.

Here, too, are to be interwoven exercises in arithmetic. Let the new arbitrary measure be measured by the old, or by the actual inch measure. Let us suppose that the former amounts to 1·2 of the latter. This means: The line which in the new measure we call one or a unit contains the inch, which in the common measure is called one or a unit, once, and, in addition, it contains two tenths of the same inch. Now, in all these triangles the radius is always unit, but this unit may be larger or smaller, the triangle becoming larger or smaller correspondingly. This we have seen in the exercises in enlarging the triangles. If a triangle, then, is to be drawn by the measure just adopted, the radius will be equal to the unit of this measure, or to one and two tenths inches. How large, then, will be, for instance, the tangent and secant of 60°? Our number for this secant is 2. This means it contains the radius exactly twice. But 1·2 taken twice is 2·4. The number for the tangent of 60° is 1·7. This means that this tangent contains the entire radius once, and, in addition, seven tenths of the same. Therefore, 1·2 must be taken 1·7 times. The computation whose signification is readily guessed, if not already known, is as follows:

$$\begin{array}{r} 1{\cdot}2 \\ 1{\cdot}7 \\ \hline 1{\cdot}2 \\ {\cdot}84 \\ \hline 2{\cdot}04 \end{array}$$

That is to say, eighty-four hundredths are seven tenths of 1·2; being added to once 1·2, they make 2·04—that is, two units, no tenths, and four hundredths.

We might find the perimeter of the triangle, or the number of inches to which all its sides amount when taken together. We need only add the radius, the tangent, and the secant:

$$\begin{array}{l} 1{\cdot}2 \\ 2{\cdot}04 \\ 2{\cdot}4 \\ \hline 5{\cdot}64 \text{ or } 5\,\tfrac{64}{100} \text{ inches.} \end{array}$$

For the same measure find the tangent and the secant of 65°:

The tangent.	The secant.
$\begin{array}{r} 1{\cdot}2 \\ 2{\cdot}1 \\ \hline 2{\cdot}4 \\ {\cdot}12 \\ \hline 2{\cdot}52 \end{array}$	$\begin{array}{r} 1{\cdot}2 \\ 2{\cdot}3 \\ \hline 2{\cdot}4 \\ {\cdot}36 \\ \hline 2{\cdot}76 \end{array}$

Find the circumference of the right-angled triangle of 65° at the assumed radius:

$$\begin{array}{l} 1{\cdot}2 \\ 2{\cdot}52 \\ 2{\cdot}76 \\ \hline 6{\cdot}48 \text{ or } 6\,\tfrac{48}{100} \text{ inches.} \end{array}$$

These computations are not yet very exact. But at this stage the purpose is only to sketch them, as it were. As knowledge grows, the computations also become more precise.

But there is a great deal in making an early acquaintance with decimal fractions, the employment of which should be introduced into every school. In the nature of the case it is impossible to work as conveniently with common fractions. Besides, decimals are likely to occur in whatsoever can possibly be termed scientific computation. Mental arithmetic might be extended to them also.

Next, let right-angled triangles be sought wherever they are likely to be found—in tables, windows, walls, houses, fields; and they are found in any rectilinear, quadrangular figure the moment it be cut diagonally. Estimate by the eye between which pair of model triangles they fall. The eye may find many aids in rectifying the estimate by a number of trials till it reaches certainty. One angle would determine the whole triangle, but with this determination there must be agreement on the part of the other angle, and again on the part of the two numbers indicating the ratios for the tangent and for the secant.

In the next place, let the pupil estimate the smallest side in common foot or inch measure, and by the previously shown reckoning let him find from it the other sides and the circumference. The benefit of this species of calculation will be felt when it is applied to objects which it would be inconvenient actually to measure—such as high rooms, houses, etc. It requires only pedagogical tact and sagacity to awaken at this early stage an agreeable astonishment at the power with which numbers reach out into distance and bring close to us what seemingly withdraws from perception. If we know how to gain lively activity in the exercises developed hitherto, some voluntary attempts will be likely to cause the children to touch the limits of their knowledge. Then the following work—and later on science—may easily assist them to break

through these boundaries. From the right-angled triangles, model forms for isosceles triangles develop easily.

From among the triangles studied thus far, let pairs of equal triangles be drawn conjoined ; first with the tangents coinciding. The two radii run into one continuous line—the base. The two secants furnish the equal sides. The altitude the former tangent indicates. The child, therefore, knows at once the base, the other sides, and the altitude in units and tenths. He also knows all the angles. The two angles at the base are equal ; that opposite the base is twice the smallest angle of the right-angled triangle used in forming the isosceles triangle. All this we can make the child find for itself under the guidance of questioning. The right-angled model triangles are 9, counting that for 85°. On the present plan there will, then, result likewise 9 isosceles triangles. To these we add 8 new ones by again placing two equal right-angled triangles together, only this time along the radii. The base is now made by the two tangents. The angle opposite the base will be obtuse. Again, all numbers for the angles and sides are known at once. The isosceles model triangles are 17 altogether. But the two largest do not easily find room upon a slate. The rest should be drawn often in different positions ; their perimeters also can be computed by a method spontaneously found from what has preceded for a variety of measures, and the triangles themselves may be detected in a variety of objects.

IV. Episodes.—Superficial Content of Triangles.—The Circle.—The Ellipse.

Pairing equal right-angled triangles by placing them together along the secants, and by inverting one of them so as to oppose tangent to tangent and radius to radius, rectangles are formed easily determined as to their square

contents, and which, because they are derived from triangles, are the best means for making obvious the mensuration of triangles and the influence of their form upon their content.

The triangles having been drawn together upon the slate so as to form rectangles, let the pupil use the square measure, Fig. 2. To avoid confusing the figure, *fe* has not been produced as far as *bc*; hence only one square inch is indicated. But the linear measure *ab* suffices plainly to show even to a child how many whole square inches have room in the rectangle under measurement. The square inch divided into hundredths serves to measure the remainder which is contained in the rectangle in addition to these wholes.

Let the first rectangle be that which originates from the first triangle, the second from the second, the third from the third, and so on. Then the first rectangle will be a square, as the tangents of 45° are equal to the radii, and hence none but equal sides are given to the quadrilateral. The second rectangle comprises that square or a whole inch, and in addition two strips as long as the square, each strip equalling nearly 10 hundredths of the square; nearly two, because the tangent of 50° is a whole and nearly two tenths. The rectangle determined by it has therefore a square content of one unit and nearly 20 hundredths. Here it is seen at once how the numbers for the rectangles depend upon those for the tangents. In the next following, or third rectangle, we get more than 1 and $\frac{40}{100}$; for the tangent of 55° is more than 1·4. The number of units is the same in both length and surface, but the number of tenths in the tangent becomes ten times as many in the surface, 2 becoming 20; 4 becoming 40; 7 becoming 70; the results, however, not being tenths but hundredths. Thus the numbers for these rectangles

are easy to memorize. Only, do not "for short" say 4 tenths for 40 hundredths. You would confuse the concept of the divisions in square measure. The square is not divided into tenths; it is divided into hundredths. It must be so divided, for the reason that each of the sides, considered as linear measure, is divided into tenths.

The rectangles increase very irregularly and with increasingly large differences. Call attention to the fact that even this arises still from the uniform progress of the angles in the original triangles. If we desired the rectangles, and hence, to begin with, the tangents, to progress uniformly, what would have to be the course of the angles? Obviously, the steps would have to become smaller and smaller; ultimately, as the rectangle grew very long, almost imperceptibly small.

Each rectangle is double the triangle whence it originates. The triangle, consequently, is half the rectangle. Thus, then, the square content of the model triangles, too, is found. We need but halve the numbers for the rectangles. The first triangle is $\frac{1}{2}$, or 50 hundredths; the second is $\frac{1}{2}$ of almost $1\frac{20}{100}$, or it is nearly $\frac{1}{2}$ and $\frac{10}{100}$, or 50 and 10—that is, 60 hundredths. The third is more than $\frac{70}{100}$, etc.

These numbers are all valid for enlarged standards. Only, it is essential first to consider accurately the enlargement of the square measure. A square must have four equal sides. They are, therefore, all four determined, if one is established. But one side of the square that is used for measuring all surfaces is as long as the line which in the linear measure we call one or a unit. This Fig. 2 shows. If this unit of linear measure is enlarged, the square, the unit of surface measure, must conform. How does it conform? If length *bc* is doubled, thus becoming *bd*, will the square of it also become only twice as

large? No doubt on the line *bd* there is room for two squares side by side, each as large as *bcef*. If we double the base *bc*, leaving the altitude *bf* unchanged, the surface determined by both will therefore also be doubled. This proposition is correct and very useful. Only, it can not here be applied. For, obviously, we have not done what we should have done. We were to enlarge the square. Instead of the smaller square, there was to be a larger square. But by the doubling there has arisen no square, but a rectangle, one of whose sides is twice as long as the other; the shape is therefore completely changed. The sides were to remain equal. Consequently, doubling one, we should have doubled the other. By being doubled, it doubles the previously originated rectangle. But this rectangle was already a doubling of the square. Hence, the square is twice doubled, or contained in the enlarged square four times. Suppose, further, that the length *bc* becomes thrice as great. By this fact alone, before the altitude is altered, the square *bcef* will be multiplied by three; and the triple square will be tripled a second time, since the height, as the sides must remain equal, is also to become three times as great. Three squares thrice make nine squares. The square becomes, therefore, nine times as large by the base becoming three times as large. If *bc* is taken five times, then the square would be multiplied by 5 twice—that is, it would be taken five times five times, or 25 times. As many times, then, as *bc* is taken, so many times, in order that the sides may remain equal, *bf* also must be taken. Hence, the square will always be multiplied twice by that number by which the side was multiplied only once. This truth remains, though we multiply by fractions. Let the fraction be $\frac{1}{2}$. Then not only is *bc* to be taken half, which would halve the square, making a piece like *bqfn*, but *bf* also must be reduced to

its half, *bp*. Thus there remains of the one half of the square only its half. The square has been halved twice, or been twice multiplied by $\frac{1}{2}$; and since one half of one half is one quarter, the whole has been changed into its fourth part, *bqpo*. Or suppose multiplication by 1·2. Then the linear measure being taken once, and in addition to it two tenths, there are added likewise to the square, before any alteration in the altitude, two tenths of it. But these tenths are strips, each containing 10 hundredths together, therefore they make 20, and, adding the square itself, 120 hundredths. Now, as the height also must be multiplied by 1·2, we have for the square once 120 hundredths, and in addition to it 2 tenths of 120 hundredths. The tenth part of 120 is 12, which being taken twice is 24; consequently, the complete result is 144 hundredths, or $1\frac{44}{100}$. This may be computed as follows:

$$\begin{array}{r} 1{\cdot}2 \\ 1{\cdot}2 \\ \hline 1{\cdot}2 \\ {\cdot}24 \\ \hline 1{\cdot}44 \end{array}$$

That is, the square is to be taken 1·2 times 1·2 times. We seek, therefore, in the first place, 1·2 times 1·2—which we found in the reckoning just shown—and next multiply the square instead of twice by 1·2 only once by 1·44. By this means the two required multiplications are accomplished simultaneously, just as instead of twice multiplying by 3 we multiply at once by 9.

Before going further this must be made absolutely plain to the children, still more in detail than here, and by a still greater number of examples. Next, let them draw an enlarged square measure on the slates, and let them combine triangles, whose radius is equal to the side of the enlarged square, into rectangles. A few di-

visional lines drawn in these rectangles will show to the senses of the child that the enlarged rectangles, and therefore the enlarged triangles, contain as much of the enlarged measure as former figures drawn by common measure contained of the latter. It will, in other words, be clear that the numbers formerly found for the rectangles are valid for any measure. Consequently, it is plain that in order at any time to find the content, we need not multiply the measure by the appropriate number, being careful only not to confound square measure with linear measure. If the latter, being one side of the square measure, is given, the square measure itself must be found from it in the first place.

The treatment of surface mensuration here adopted, separating measure from number more carefully than is done by the usual way of multiplying the base by the height, has one advantage which is important for the A B C of Sense-Perception. The children become habituated to separating mentally the size and the shape even in the case of surfaces, and learn to think of drawings presented to them merely as symbols for objects which may be greater or smaller. They learn to conceive any form as an alteration of some other form, and of the numbers which serve to distinguish different forms as abstract ideas of ratio. In the measurement of actual objects, however, where the purpose is not to train the intellect but to obtain the sought-for result as quickly as possible, multiplication of the base by the altitude is a far shorter way than it would be to find first merely the form by the comparison of the angles at the diagonals, and then the enlargement of the measure by estimating the shortest side. The teacher will act accordingly in the mensuration of fields, windows, houses, and so on. He can not expend too much time or care upon the perfect

conversion of percepts into concepts. But, to facilitate work which may be necessary in later life, he may afterward show that two numbers belong to height. One determines it as tangent of the angle at the diagonal. The other, which is owing to the enlargement of the measuring square, is common to it and to the base line. In the method of considering the subject which we have adopted these two numbers are separated, as only the first depends on the form, the second depending on size. The separation, however, is not serviceable for the determination of the square contents, since in this the two numbers recombine. Consequently, the altitude might have been given, like the base, by a single number. This would have contained both those numbers. To it must be joined multiplicatively the number for the base, to give as result the content.

Small but perceptible instances of incorrectness in the attempts at actual surface mensuration will again, and more strikingly than formerly, remind the pupil that the numbers for the tangents are known not accurately but only as far as tenths. There is no harm if the beginners become impatient at it. This impatience is a purposely excited desire for knowledge. Remind them that they found these numbers by their own measurements. Suggest to them that they measure more accurately if they can. Promise for the future the aid of the science in whose power it is to satisfy every wish of this kind.

Adopt a like course in the case of very oblong quadrangles. Here the differences of the tangents appearing in the table become so large that the table numbers cease to determine anything exactly. Suggest to your students that they find the tangents for angles differing by less than five degrees by measurements similar to the first. When the slate becomes too small for the purpose, lines

may be drawn on smooth ground in the open air, or indicated by stakes and strings. These lines may be measured by feet and tenths, or even hundredths of feet, much depending, to be sure, upon how large and how accurately at the start the angular measure has been inscribed on the ground, and how correctly the perpendicular has been erected on the radius.

The following is such an addition to the table of tangents and secants:

The tangent	of	78°	is	4·7;	the	secant	is	4·8.
"	"	" 83°	"	8·1;	"	"	"	8·2.
"	"	" 88°	"	28·6;	"	"	"	28·6.

The preceding work may easily be extended to oblique quadrangles and all species of triangles, and hence to the whole subject of surface measurement.

In the first place, we may again join two equal right-angled model triangles along either their tangents or radii, but reversely, so as to form rhomboids. Obviously these are equal in content to the rectangles and the isosceles triangles originated by joining the same right-angled triangles. Again, at one side of rectangles of every kind, cut off an arbitrary triangular piece; only, the cut must be straight and pass through a vertex of the rectangle. The piece can be added on the other side. The originating oblique quadrangles, by similar cutting off and joining on, will become still more oblique, so as to make it possible in this way to run through all the varieties of rhomboids. In doing this, nothing is added to or taken from the surface of the original rectangles. Only the position of the parts is altered. Thus it is easy to make plain the proposition that any parallelogram may be changed into a rectangle of equal content at equal base and altitude, since these also are not altered in the cutting. In other words, the square contents of the rectan-

gle indicates the square contents of the rhomboid. Triangles being halves of rhomboids are therefore halves also of the corresponding rectangles, and may be computed as such. To conclude, any figure can be divided into triangles whose sum will equal the square contents of the figure. It is not necessary to go into details on the subject, for we have already struck into the path of discovering geometry for ourselves.

In this episode may be included also, for the ablest among the pupils, a preliminary determination of the circle. First, lead them to observe that in the smallest parts of the circumference the curvature becomes unnoticeable; that a small arc is almost equal to its tangent. On this occasion the word tangent is completely reinstated in its usual mathematical significance. Next, if necessary by the aid of the ruler and the protractor, have them draw a line of 10 inches, and join to it a line at an angle of 10°, and at the other end of the line draw a perpendicular. The latter when cut off by the two lines forming the angle will be the tangent of 10°, and hence only a very little more than the arc of 10° for a radius of 10 inches. Of such arcs there pertain to the whole circumference 36; measured by the horn-leaves, the tangent of 10° will be found to be 1 and about $7\frac{1}{2}$ tenths inches in length. We accordingly multiply 1·75 by 36:

$$\begin{array}{r} 1{\cdot}75 \\ 36\phantom{{\cdot}7} \\ \hline 10{\cdot}50 \\ 52{\cdot}5 \\ \hline 63{\cdot}00 \end{array}$$

The circumference for a radius of 10 inches would be therefore about 63 inches. Conceive the circle ten times as small. There results for the radius of 1 inch a circumference of 6·3 inches. It is customary, however, to com-

pare the radius with the half circumference, or the diameter with the whole circumference. One half of 6·3 is 3·15, a number which is a little too large, as we computed it on the tangent instead of on the arc. It should really be a little more than 3·14.

From this point on it is again only necessary to present geometry comprehensibly in order to make the strictly geometrical transition from the circumference of the circle to its square content.

Pestalozzi has included in his A B C of Sense-Perception the ellipse, or, as he incorrectly terms it, the oval. It deserves to be included, partly because it accustoms the youthful draughtsman to a changing curvature, partly because we need it so often when circles not directly facing the eye are to be drawn. Under such circumstances the ellipse is usually drawn wrongly. It is well to make sure of its course by a rule, even though by the analytical investigation of this line we can in this place derive nothing but just the rule.

Draw a circle, and in it a horizontal and a perpendicular diameter. Exactly parallel to the latter draw a few lines through the circle. On either side of the horizontal diameter, between it and the circumference, divide these lines in half, or cut off from them $\frac{2}{3}$, $\frac{3}{4}$, any fraction you please, only use the same fraction for each line. Through the points by which you indicate the fraction draw a line. It will be a regular ellipse. Its form will be determined by the chosen fraction. It may be altered by changing the latter. Not till after these preliminary exercises is permission given to draw ellipses freehand.

V. A General View of all Triangular Forms.

Our preparations are now complete for presenting in one view to the imagination and the understanding

every variety of triangular ground forms. The first mechanical occupation upon lines and circles was introduced among the first beginnings of instruction. It seemed only to furnish play for the hand. The matter became a more serious occupation when the child was required accurately to measure and discriminate lengths and angles. The right-angled triangles even caused us, if not to gain an insight into a scientific necessity, at least to feel it and to find it; for their angles and sides revealed a system in which there was mutual determining and being determined. This important step in the training of the understanding, which required a close and collected reflection, was rewarded by affording an opportunity for observations on objects that illustrate those principles, and which could be followed up with freedom to a greater or less extent. In order to secure this liberty let the teacher discontinue the regular lessons in this branch. Having included as much of the Episode as he deems well for his pupils, let him pause for a few weeks, introducing during the periods which were devoted to the A B C of Sense-Perception some other instruction. It is well to let recollection slumber a brief while, that it may reawaken with uniform vivacity, so as to have the distinction disappear between that which was learned earlier and that which was learned later, and so that all the details may be properly adjusted.

When the thread is taken up again, give a synopsis of the preceding exercises. Next, invite the pupil to consider the question whether it is likely to be the intention of this instruction to determine only right-angled forms. Even if one chooses to evade greater difficulties by occupying himself with only the simplest forms, namely, those which consist of spaces inclosed by three lines, the three lines are not always joined into right-angled triangles, but

into a very great variety of triangles. The tangent, which, by the gradual opening of the opposite angle, was cut off at greater and greater length in ever-varying proportions, would be cut off in a still greater multiplicity of ways should we incline it more or less to either side. For every angle of inclination that we should give it, there would be possible an entire series of model triangles like our right-angled triangles. A host of possible triangles are revealed, any one of which may be expected to be formed by us. For example, opening a door, you will change the direction of its base line to every point in the room. Mention the case of a man going straight ahead or to one side while passing two trees, etc.

It is possible to find even more interesting examples. They are not to be despised. But the important object is to excite in this connection a kind of speculative interest that may, to be sure, be spiced with little accessory interests, but that neither can be thus replaced nor must be thus overbalanced.

The question then arises how we shall traverse the multiplicity of triangles that extend on all sides. The observation that a perpendicular may be dropped in any triangle from the apex to the opposite side, dividing the triangle into two right-angled triangles, offers an easy means of orientation. Thus reduced, any triangle may be looked upon as composed of two right-angled triangles. Hence, there are as many possible triangles as there are possible pairs of right-angled triangles. To set forth, therefore, from among the multitude a sufficient number of model forms between which the others must lie, we combine our right-angled model triangles into as many pairs as possible. The measurements and computations which we have made upon the right-angled triangles will thus furnish the basis for the computation of all the ratios that

are necessary to determine the forms of the remaining triangles. The way is now clear. Patient diligence alone is needed to follow out the road to the end.

The perpendicular dividing the triangle into two right-angled triangles divides the angle at the apex into two parts. Each part determines perfectly the form of the right-angled triangle on its side. For this reason, the possible number of ways in which the angle at the apex can be composed of these two parts fixes the number of possible combinations of right-angled triangles.

Across the slate draw a horizontal; erect a perpendicular upon the middle, not too high. At the upper end on either side of the perpendicular imagine angles opening gradually. When each angle reaches 90°, both the external sides will run parallel to the horizontal and into one continuous line. Until they do, they and the horizontal together inclose triangles ever varying. All these triangles would be isosceles if the angle opened equally on both sides every time. If, however, we desire to have every possible triangle, then, for each magnitude of one angle, the other must traverse every possible magnitude, from 0 to 90°. The angle at the apex of the entire triangle, which is the sum of these two angles, will thus be made up of two parts in every possible way.

Let the angle at the left of the perpendicular have at first 5°. To begin with, let that angle remain unaltered. Let the angle at the right of the perpendicular have at first 5°, then 10°, then 15°, then 20°, and so on up to 85°. Next, give to the left angle 10°, and again leave it unaltered, while the angle to the right begins once more at 5° and grows by steps of 5°. Next, increasing the angle at the left to 15°, keep it so, letting the angle at the right again begin at 5°, and run through 10°, 15°, 20°, and so on. Then assign to the left angle 20°, and let the angle

at the right traverse its series. It is obvious how this continues.

It is certain that we should thus get all the possible combinations of right-angled triangles. But there would be among them such as would differ in nothing but the reversed position. For instance, right at the beginning, when the left angle has 10°, and the angle at the right commences at 5°, the combination is repeated that occurred when the angle at the left was 5° and that at the right 10°. Only we have changed from left to right. Such a difference contributes no new form. It is here superfluous.

To eliminate transpositions and leave the combinations that are of exclusive importance regarding form, observe that there will be a transposition every time the angle at the left is greater than the angle at the right; for we have had this greater angle once before, at the right, when that at the left had grown as large only as the smaller angle at the right is now. For example, suppose we have at the left an angle of 35°, at the right one of 25°; then there must earlier have been an angle of 25° at the left, and since at that time the angle at the right traversed its series it came to 35°, and at that time represented the combination. It is now repeated, only in reversed position.

Therefore let us guard against ever having the greater angle at the left.

It will not happen if the angle at the right each time begins not from the beginning but from the number of degrees which the angle at the left has already at the moment. For instance, if the latter has 75°, the former traverses only 75°, 80°, 85°.

Now recall that in each of the right-angled triangles which are formed at the two sides of the perpendicular,

the angle at the base is determined by the angle at the apex. As the angle at the perpendicular opens, that at the base must become smaller. If the one increases by 5°, the other loses as much. One goes from 5° to 85°, consequently the other from 85° to 5°. This occurs on each side of the perpendicular. The angle at the apex is always the sum of the two angles at the perpendicular.

These reflections are the foundation for the first table which is appended to this book, and which the pupil under the teacher's directions must make up for himself. Above runs a row of numbers from 10 to 90. Under each number is a line drawn straight down. It represents the perpendicular which divides the angle at the apex, the upper number, into two angles, right and left, noted by smaller figures on each side of the line. This determines the two angles at the base, which are indicated by large figures under the smaller. The first triangle at the left will have therefore at the apex an angle of 10°, below, at the base, two equal angles of 85°. Thus, throughout the table large figures arranged by threes triangularly about a stroke indicate the three angles of a triangle; the small figures written between them indicate the division of the upper number which stands for the angle at the apex. The uppermost horizontal row of triangles may be called the first, the next under it the second, the following the third, fourth, and so on. In the first row the angle at the left of the perpendicular remains 5° and consequently the complementary angle at the base 85°; in the second row the former angle becomes 10°, the latter 80; the row begins one place later, as the angle at the right must not again become 5°, since this would be less than 10°. On the same principle the rows continue to shorten. It will be seen that this conforms with the above requirements. Carrying the eye straight down the table, we have col-

umns. The first of these on the left, which contains only one triangle, may be called the first column; the next, which contains two, the second column, etc. In each column the angle at the right of the perpendicular, and consequently that at the base, remain constant. It is for the sake of this regularity that the rows began later and later. Finally, traversing the table diagonally from the right to the left below, the angle at the apex remains constant on any diagonal, because this angle is the sum of the two angles at the perpendicular, one of which grows by as much as the other decreases, since by taking a diagonal we simultaneously traverse the columns toward the left, while traversing the rows in a downward direction.

The middlemost diagonal divides the table into two equal but dissimilar halves. It runs through all the right-angled triangles—those familiar model triangles. On its left are the acute-angled, on its right the obtuse-angled triangles. At first sight the multiplicity at the one hand seems as great as at the other. But the number of acute-angled triangles can be reduced very much after the following reflections.

In each of the triangles the angle at the apex is divided into two parts. But which is the angle at the apex? In right-angled and obtuse-angled triangles, no doubt, it is the right or obtuse angle; for attempting a perpendicular to the base from any other corner of the triangle, the perpendicular drops outside; it could not meet the side opposite unless the latter were produced. This case does not belong here, as the angle at the apex would not be the sum of the two angles at the perpendicular. Therefore, in right- or obtuse-angled triangles, so far as our subject is concerned, the largest side is always the base, and the largest angle opposite to it is at the apex. But in acute-

angled triangles no such decision mark is furnished. Turn it as we please, any side as base has an angle hovering above it, whence the perpendicular falls within the triangle. Here we have a threefold basis of choice, except in isosceles triangles, where the choice is only twofold, since we can not distinguish which of the equal sides is taken as the base, the sides themselves differing in nothing. Now, since the table contains all the possible cases, every isosceles triangle occurs twice, and all the other triangles occur three times—that is to say, in the half which contains the acute-angled triangles. In the other triangles no repetition occurs.

The front or left half of the table must therefore be searched more narrowly in order to find the repetitions.

They easily show themselves if the student has a thorough hold on what has preceded. Look at the uppermost row: in it the angle of 85° remains constant; the other two approach each other in opposite but equal sequence, the one going from 10° to 85°, the other from 85° to 10°. Necessarily, one half the row repeats the other. Furthermore, from the last triangle but one in the first row, go down parallel to the diagonal of the right-angled triangles. Compare each triangle you reach with the uppermost triangle of its column. These, too, repeat. For in the diagonal, as has been observed, the angle at the apex remains constant. It has here 85°. It is the same, therefore, as the angle at the left in the first row, which also remains at 85°. In addition to this, in any given column the angle at the right is constant. But if in two triangles two angles are equal, the third angle must be equal also, as the three together equal 180°. This will be understood at once, as, in combining the two right-angled triangles to make a single triangle, the latter gets all the acute angles of the two, hence twice 90°. Thus, this geometrical propo-

sition, which, of course, may also precede, can easily be rendered plain from the preceding.

What is here shown in respect to the first row and its appertaining diagonal may be easily enlarged so as to apply to the rest.

At the front each row begins with an isosceles triangle. This follows from the whole arrangement of the table, for the angle at the right is not to be smaller than that at the left. We commence it, therefore, when it equals the angle at the left. Now, the angle at the apex grows, that at the base on the right decreases. Thus, approaching each other with equal steps, either of them must at some time get to the place where the other began. This repeats the first isosceles triangle. At this moment cut off the row. Then one half of the cut-off is the repetition of the other, only in inverse order. This will be shown with the greatest plainness by the table itself, which during these explanations should be kept before the eye constantly. From the point where the row was cut off go down diagonally. In the diagonal the angle at the apex remains constant. But in the isosceles triangle, from which we go down, this angle at the apex equals the left angle at the base, which angle remains constant throughout the row. Thus the row and the diagonal have one angle in common. In addition, both intersect the same columns where they always strike the same angles. Thus they have the second angle, and consequently all three angles, in common; hence, their triangles are always the same.

The above conclusions presupposed from the first that the isosceles triangle commencing each row has at the apex a smaller angle than at the right at the base. Then these angles approached, the smaller increasing, the larger decreasing. But the presupposition holds only as far as

the sixth row, which begins with the equilateral triangle. Further than this row, therefore, the conclusions will not hold. But this is exactly far enough to cover the whole half of the table. This, again, a glance at the table will make obvious.

The teacher will have some trouble in making plain to the children the general view of all possible triangles. But his pains will be rewarded by the inestimable advantage of accustoming pupils to traverse completely and with steady eye a whole field of concepts. This combinatory exercise of the understanding is indispensable in teaching, for it is unusual for even cultured minds to have a spontaneous mastery of it. Teachers, on the other hand, recommend for the discipline of the understanding many things which are trivial, and even some things which are contrary to the purpose; the latter being things in which the student must succeed untaught if he is not to become pedantic. Should the teacher feel that the survey of the table is none too easy for himself, let him not infer that "the children can not learn this," but let him consider how much must have been lacking in the instruction which he received in his own youth. Practice in combining should be an absolutely essential portion of every cycle of instruction. Many sciences would be better off if their founders and cultivators had possessed it. And many things in the lower grades, especially among others declension and conjugation, would no longer deaden the spirit but enliven it, and would be conceived of far more swiftly, easily, and surely if on the occasion combinatory reflections were introduced. The first and the second assertions are both so safe that even a writer quite without weight and reputation may risk throwing them out without proof, with full confidence that the course of science and of pedagogy will some time demonstrate them for him.

In addition to these considerations the concept of the triangle is a concept of such extreme importance as to deserve in the highest degree the pains of following it out completely through all its modifications. Given a good instructor, the only difficulty he can meet in doing so is that the attention of his pupils will tire too early. But if they do not lack the power of attention altogether, if the teacher is not too weak to ask of them that degree of exertion which they can easily bear, and that persistence through an hour or a few hours which, after an equivalent of as much time for recreation, even a boy must become accustomed to in order to be able to accomplish and perfect anything, then no harm is done even if it should be impossible to unwind before the boy the whole thread of those considerations at once and unbroken. Allow time for recovery from fatigue. Without disadvantage some of the following computations may be interspersed. A few days later recommence, changing somewhat the presentation. Analyze, explain, objectify every single conclusion most accurately, and permit neither yourself nor the children to lose patience until a perfect insight is apparent.

VI. Computation of Sides.

In the field which we have now surveyed we must next contemplate and reflect upon the details—that is to say, they must be computed.

Two auxiliaries are serviceable for properly apperceiving every triangular form. First, before computing any triangle, let the children sketch it in miniature on the slates. Second, to avoid habituating the imagination exclusively to small drawings, have an instrument for presenting forthwith upon a large scale any triangle with which you happen to be occupied.

The following arrangement has been tried for such an instrument, which in any event will resemble the triquetrum of the ancients, and the model has turned out well for handy use :

A staff of wood, *ab*, Fig. 4, is joined at *b* to another staff, *be*. On the other side, at *a*, there is another joint, which, however, is not fastened to the staff *cd* immediately. Instead, in the inner vertical side of the staff *cd*, a side not shown in the figure, a groove runs widening inward. In this groove moves a small travelling piece, indicated in the figure by a dotted line between *m* and *n*. The traveller is connected with the staff *ab* by the joint at *a*. Hence, *ab* may be moved on *cd* ; also the angle made by these two staves can at pleasure be closed and can be opened to 90°. *ab* and *eb* may be opened to 180°, and may be closed as far as the point *d* of the staff *cd* allows at any position of the instrument. *cd* and *be* must be at the least five times as long as *ab* ; it would be better to make them longer still but for the instruments becoming unhandy. The two protractors seen at *a* and *b* would best consist of very small brass arcs ; they may, however, be made of horn-leaves. They are fastened to the staff *ab* ; the other two staves must be free to turn under them. A third protractor may be applied at *d* to the staff *cd* ; it need not have more than 60°. *cd* and *be* are, in the first place, divided into five equal parts, each equal to the length *ab*. Furthermore, *ab*, although in the figure in order to prevent confusion this is not indicated, is divided into ten parts, as is also each of the five units on the staves *cd* and *be*—that is to say, *ab* is here taken as the unit, and we mark units and tenths on the other staves. If the instrument is not too small, hundredths may be marked likewise. If it is, they must be estimated by the eye as accurately as possible. The larger the instrument is made the

better. There ought really to be some arrangement for allowing the instrument to execute its motions upon the wall. By this means the triangles would be most obvious to the eye. If the teacher prefers to have it at his side on the table, he will get a convenient size by making *ab* 4 inches long.* The instrument may be made by any good joiner at a small cost. Only, you will be obliged to draw the protractors yourself on horn-leaves; and especially you will be obliged to nail them on yourself, for this requires the utmost accuracy, such as one can expect of no artisan. Unless the centres of the protractors are exactly on the points *a*, *b*, and *d*, and unless their base lines coincide with the edges of the staves to a hair's breadth, the instrument is useless. Protractors of horn-leaf are difficult to glue on; still less can they be fastened with ordinary nails, which split the horn-leaf. The best expedient is to use common sewing needles, breaking off the upper half; the remaining half can be driven in with a tuning hammer. Two will hold the horn-leaf quite firmly, but a few additional ones may be driven in by way of precaution. If the instrument is to show the triangles quite accurately, it would better be set by the numbers in the second table, for the determination of a triangle by the sides is always more accurate than by the angles. This remark should not be turned as an objection against the method here chosen of finding the sides from the angles. By the angles the form is immediately offered to sense-perception. Besides, these preliminary exercises have not as yet, as mathematics itself has, the means that can be used for finding the angles from the given sides. Further-

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* For the purpose of stronger objectification, have the whole black, except the inner edges of the staves, which strictly show the triangle, and which should be white or red.

more, if we had assumed rational sides—and to do so is possible for the three sides simultaneously in very rare instances only—the angles would have turned out irrational —a concept having so little sensuous clearness as to be utterly unsuitable for an A B C of Sense-Perception.

In the computation of the sides, which we are about to enter upon, proportion is both necessary and, which is no small advantage, it is objectified.

There being even to-day some adults who complain that the reasons for proportion were never clearly presented to them, it may be permitted for our A B C of Sense-Perception at this point to make an incursion on the A B C of arithmetic.

If you wish to buy three yards of cloth you will inquire the price of one yard, and conclude: Since I wish to buy one yard three times, I shall have to pay the price of one yard three times. Put generally—as many times as I order one yard, so many times do I pay the price. For computation, this extremely simple illustration is set down thus:

$$1 : 3 = b : 3b,$$

b signifying the price. Thus expressed mathematically it reads: One yard increases to three yards, hence the price b increases to three times b. If the price were $4, we should read: One yard increases to three yards, hence $4 increases to three times $4, or $12.

In the case of so easy an example no computation will be gone into. Rather, perhaps, for the following:

$$100 : 850 = 5 : \frac{850 \times 5}{100}$$

read thus: $100 increase to $850, hence $5 annual interest increase to 850 times $5 divided by 100. It is understood that the supposition is that one wishes to know how much interest will be yielded by a capital of

$850 at 5 per cent annually. As many times as one lends $100, so many times does one receive $5 annual interest. Now, how many times have $100 been loaned? You can not tell in this case without a fraction. It is convenient to think the subject over in the following form: Suppose that on every single dollar loaned I were to receive $5; then for $850 loaned I should obviously receive 850 times $5. But by actually making this demand on the debtor, it is certain one would ask a hundred times as much as he would pay. It is therefore correct to demand the one hundredth part of 850 times $5. Hence we shall divide 850 times 5, or, what is the same thing, 5 times 850 by 100. The computation will be as follows:

$$\begin{array}{r} 850 \\ 5 \\ \hline 4250 \end{array}$$

it being only necessary to write 42·5 instead of 4250 to finish. For a division of 100 contains two divisions by 10; now, every figure signifies ten times less when put one place farther to the right; the division by 100 is consequently accomplished by moving each figure two places farther to the right. Thus 50 units become 5 tenths; 200 units, 2 units; 4000 units, 40 units; consequently, 4250 become 42·5—greater detail would not here be to the purpose. The following computations, however, will contribute clearness. Still, if one be not afraid of a slight trace of algebra, all that essentially belongs to proportion is very briefly said in the following formula:

$$a : ma = b : mb$$

a and b signify things known to be increased at the same ratio. Therefore, a increases to m times a, as b to m times b. Now, since it is usually impossible to state the number m without a fraction, m times b is found by taking

b am, times, and, having taken it *a* times too many, dividing *mab* by *a*. Therefore the following expression shows the entire computation, sometimes fancied to be difficult:

$$\frac{amb}{a}$$

In order now to determine the forms of triangles by the ratios of their sides, and in order to be able to compare these ratios among themselves, we must first eliminate from form the accidental difference of magnitude. Hence it is necessary for all triangles to have one side of common magnitude. Then the differences of the other sides will render the differences of form noticeable.

But since the one equal side will be for all of them the equal standard, all the other sides being determined by stating how many units and tenths of that side they contain, the question is, Which side is best fitted to be the common standard? Most naturally the least, since one measures the greater by the less. Therefore, the least side itself serving as the measure, it contains the measure neither more nor less than once. Its number is always 1. The number of any other side, on the contrary, is always larger than 1.

For the purpose of computing the sides, which computation depends altogether upon the ratios for the right-angled triangles, it is necessary for mathematics to come to our aid by a few figures determining those ratios more accurately. Were we to compute the numbers now to be found only as far as tenths, very many of the triangles thus determined would not be visibly different; for they are equal up to the tenths. They do not begin to differ from each other until hundredths are reached. Hence it is necessary for the numbers for right-angled triangles to be known at the least up to the hundredths. Even so,

considerable uncertainty now and then remains in the computation. This obliges us carefully to think over the differences of triangles in order to choose from among several possible methods of computation that one which, in every individual case, leads to our goal most surely. If, at length, in a few rare cases, even this precaution proves insufficient, then the A B C of Sense-Perception refers, by the very fact itself, to mathematics—to the science toward which the beginner's exertions are directed. The hundredths that we beg of the science as a preliminary gift will be found in the following table :

Of 45°	the tangent is		1·00 ;	the secant	more than	1·41.
" 50°	" "	" over	1·19 ;	" "	" "	1·55.
" 55°	" "	" "	1·428 ;	" "	" "	1·74.
" 60°	" "	" "	1·73 ;	" "	exactly	2·00.
" 65°	" "	" "	2·14 ;	" "	more than	2·36.
" 70°	" "	" "	2·74 ;	" "	" "	2·92
" 75°	" "	" "	3·73 ;	" "	" "	3·86.
" 80°	" "	" "	5·67 ;	" "	" "	5·75.
" 85°	" "	" "	11·43 ;	" "	" "	11·47.

Even by adding the hundredths the tangents and secants are not perfectly determined. There are lacking thousandths, tens of thousandths, etc. For the tangent of 55° we have added the thousandths, since 8 thousandths make almost one hundredth, and in the case of so small a tangent as this, so large a number of thousandths is not to be held insignificant; for it is plain that, by comparison with the entire remaining magnitude of this tangent determined by 1·42, the additional 8 thousandths amount to more than if, for example—as is actually the case—as many of them be omitted after the numbers for the secant of 80°. In connection with 5 units, a few thousandths more or less may be overlooked more readily than in connection with one unit. If one does not desire to be particularly accurate in the reckoning,

1·43 may be written instead of 1·428, but the error resulting at the end of the computation may easily amount to more than one hundredth.

The tangents and secants for 78°, 83°, and 88°, if also required, will be as follows:

78°,	tangent	4·70;	secant	4·81.
83°,	"	8·14;	"	8·20.
88°,	"	28·63;	"	28·65.

Of the actual computation of triangles* we shall give an idea, in the first place, by two easy examples (see Figs. 3 and 5). In Fig. 3 the whole triangle *abc* consists of the

* If teachers versed in mathematics desire to instruct on the basis of the present book, the following proposition of a reviewer may, under certain circumstances, be commendable: "The reviewer would use the names radius and tangent as they are used in trigonometry, and before going on to the general subject of triangles he would by the use of arithmetic, which at any rate must begin here, presenting what in relation to one angle was the tangent, in relation to the other as the radius, and conversely, derive from the ratios of the sides in the first nine model triangles that are known already, the corresponding numbers for the remaining possible nine, thus causing the fundamental table to become complete from within itself without an additional burden to the memory. It is the same computation"—not quite—"into which the author later on in the composition of triangles is anyhow obliged to enter every moment. But it is only here scientifically in the right place. Besides, it offers a considerable advantage by all the rules for the computation of triangles in general coinciding in one, thus enabling the student to carry out the development of the whole more easily and systematically." This is the language of the mathematician, seeking everywhere the formula of greatest universality. The teacher, on the contrary, avoids intentionally the mechanism of doing a great amount of work according to one rule. For this reason, and on account of other minor considerations, the proposition is not adopted in the present edition. But for pupils who themselves have an inclination to see at once through the manifold as subordinated to the general, and therefore to carry out through all special cases the

right-angled triangles *acd* and *bcd*. In *acd*, *cd*, being the smallest side, is to be taken for the radius ; *ad*, the middle, for the tangent. But in *bcd*, *cd* is the tangent, *db* representing the radius. The circumstance that *cd* in one triangle is to be considered as radius, in the other as tangent, brings in proportion. In Fig. 5 it is otherwise : *ad* is the radius for both triangles; here, therefore, we do not need proportion. Merely put down the numbers for both right-angled sections; add *bd* to *dc* ; take care to give the number 1 to the smallest side, *ac* ; and triangle Fig. 5 is computed. Being the easiest computation, it is shown first. Let the angle at the left of the perpendicular be 60°, that on the right 50°. Then the whole angle at the apex will have 110°, the angle at the right of the base 40°, that at the left 30° ; and the triangle is found in the first table, twelfth column, tenth row, except that the right and left are interchanged, which for computation is as indifferent as for the triangle itself. The numbers demanded by the angles are, in *abd*, 1, 1·73, 2 ; in *acd*, 1, 1·19, 1·55. But in order to get rid of all confusion which might possibly be caused by the decimal fractions, imagine the entire triangle and all its sides 100 times as large; then the numbers become :

for *abd*,	for *acd*,
100, 173, 200 ;	100, 119, 155.

Now, the line *bc* consists of the two tangents *bd* and *dc*; therefore, add 173 and 119, making 292. The number 100, pertaining to the radius *ad* of the two right-angled triangles, is now superfluous, for in the whole triangle *ad* does not occur. But *ac* is the smallest side ; therefore, ac-

general rule to completion, the more uniform mode of computation would have its advantages, although the results are somewhat less accurate, as an attentive comparison will show.

cording to the preceding, its number 155 must be change into 1. But if this side, or its number, rather, become 155 times smaller, then, in order not to destroy the form everything in the triangle must be diminished as man times: *ac* was 155, *ab* was 200, *bc* was 292; all these numbers must be divided by 155. 155 ÷ 155 is of course 1 the following divisions are expressed thus:

155)200(1·29	155)292(1·88
155	155
450	1370
310	1240
1400	1300
1395	1240
5	60

These divisions are continued into decimal fractions Let the remainder 45 in the third line be thought of a 450 tenths; in these are contained 2 tenths of 155; then let the remainder 140 in the fifth line, being tenths already, be thought of as 1400 hundredths; in these are contained 9 hundredths of 155. As one tenth of 155 wil be 15·5, one hundredth of 155 will be 1·55; two tenths therefore, twice 15·5 or 31·0, and nine hundredths of the same number are 13·95. With these tenths and hundredths, as is apparent, we have continued the reckoning as though by whole numbers, only annexing to every remainder a zero. The zero is not illegitimately to make the remainder ten times as large, but only to represent the remaining units as ten times as large a number of tenths, the remaining tenths as ten times as large a number of hundredths. The division of the tenths produces obviously tenths, the division of the hundredths produces hundredths.

In both divisions the number of the hundredths has

become a little too large, but the mistake in the first division does not amount to half a hundredth, and in the second to scarce a thousandth. The errors originate because the numbers for the tangents and secants are not determined with sufficient accuracy even by the thousandths which we annexed. In the second table appended to the book, and computed by the aid of logarithms to tens of thousandths, the numbers indicating the ratios for the two largest sides of this triangle are given as 1·2855, and 1·8794. The smallest side being always 1, it is everywhere omitted from the table.

If the least obscurity should still remain as to how these numbers can determine the sides, the use of the instrument previously described, setting before the eye immediately the units, tenths, and hundredths, will secure the highest degree of clearness.

Triangle Fig. 3, at the apex of which the angle of 85° is divided into 50° and 35°, is found in the tenth column, seventh row. Again put down, in the first place, the radii, tangents, and secants for both the right-angled pieces:

50°	55 °
100, 119, 155	100, 143, 174.

Here *cd* in the triangle *acd* is 100, but in triangle *cdb* 143. The reason is that in the latter triangle the standard, namely, the radius *db*, is smaller than in the former, where *cd* is itself the radius. If the exercises in enlarging and reducing right-angled model triangles have been performed with care, all this should be plain. Being measured by a different standard, *ad* and *db* can not, of course, be added. It must be done, however, in order to obtain the side *ab* for the whole triangle: *cb*, being obviously smaller than *ac*, would likewise be expressed incorrectly by the number 174, if *ac* retained the number 155.

In a given magnitude a smaller measure is contained more times than a larger. Therefore, the measure decreasing, the number grows. But the measure for *acd* becomes smaller, even though we now measure the sides of this triangle by the same measure as *cdb*—namely, by the radius *db*. Hence, the number for *cd* has already grown larger; it has increased from 100 to 143. The numbers 119 and 155 must now increase similarly, since for all the sides of *acd* the same diminution of measure prevails.

Suppose the 100 had increased to 200, it would have grown to twice its former size; hence, 119 and 155 also would require to be doubled. Or suppose 100 had increased to 150; it would have grown one and one half times as large as it was before. Therefore, 119 and 155 also would have to be taken one and one half times. Now, 100 has not increased to quite 150. How many times as large as it was it has become, can not be indicated without a fraction; but one thing is certain, namely, that 119 and 155 must be taken just as many times. Now, remember what we have said of proportion, and you have the following computation:

$$100 : 143 = 119 : \frac{143 \times 119}{100} \qquad 100 : 143 = 155 : \frac{143 \times 155}{100}$$

$$\begin{array}{r} 143 \\ 119 \\ \hline 1287 \\ 143 \\ 143 \\ \hline 170{\cdot}17 \end{array} \qquad\qquad \begin{array}{r} 143 \\ 155 \\ \hline 715 \\ 715 \\ 143 \\ \hline 221{\cdot}65 \end{array}$$

The decimal point here performs the division by 100, as was shown above. We have next to add *db* to *ad*, or 100 to 170·17. This makes 270·17. *ac* by our computa-

tion is 221·65; *cb* having retained its original measure, remains at 174. But *cb* is the smallest side, and therefore is to become 1. As all numbers have to be diminished an equal number of times, divide 270·17 and 221·65 by 174.

174)270·17(1·55	174)221·65(1·27
174	174
961	476
870	348
917	1285
870	1218

The 961 in the third line are tenths; the 917 in the fifth line are hundredths; dividing the tenths, we get tenths; dividing the hundredths, we get hundredths.

The first resulting number is too great by not quite a thousandth; the second coincides exactly with the second table, in which the ratios for this triangle are given as 1·5498 and 1·2743, the least side being again understood as 1.

The computation of every triangle will take a course similar to these examples; but modifications are useful, that we may everywhere operate to the greatest possible advantage. To be sure, the advantage here consists at the most in a few hundredths. But he who would learn computation must deem worthy of increased care and sharpened reflection even fractions far smaller. Besides, it is possible only by the modifications to remove tedium from these exercises. What right have we to require children to repeat the identical monotonous computation as many times as there are triangles, that with persistent attention must be contemplated and reflected upon by them? And in spite of all tables, what a blurred mixture these triangles would be to them were there no differences to observe in order that the computation, in the form which the teacher requires, may be furnished!

For this purpose triangles fall into four classes. See the first table. Those obtuse-angled triangles, in which both angles at the perpendicular either equal or exceed 45°, make one class. The second contains those in which one angle at the perpendicular is less than 45°. Of the acute-angled triangles, those belong in a third class, having one angle, at any rate, at the perpendicular which equals or exceeds 45°. The fourth class comprises those in which neither angle at the perpendicular attains 45°. The last class opposes the most difficulty to computation, but is, on the other hand, the least numerous.

The first class comprises the largest number of triangles, but they are the most easily and, on the whole, the most accurately computable.

The operation required by the first class was completely shown in the example in Fig. 5. To allow the computation to be viewed free from interpolated remarks, we give an additional example. Let it be the triangle in the first table, sixteenth column, fifteenth row:

$$\begin{array}{r|l} 75 & 80 \\ 100,\ 373,\ 386 & 100,\ 567,\ 575 \end{array}$$

$$\begin{array}{r} 373 \\ \underline{567} \\ 940 \end{array}$$

$$\begin{array}{l} 386)940(2{\cdot}43 \\ \quad\ \underline{772} \\ \quad\ 1680 \\ \quad\ \underline{1544} \\ \qquad 1360 \\ \qquad \underline{1158} \\ \qquad\ \ 202 \end{array} \qquad\qquad \begin{array}{l} 386)575(1{\cdot}49 \\ \quad\ \underline{386} \\ \quad\ 1890 \\ \quad\ \underline{1544} \\ \qquad 3460 \\ \qquad \underline{3474} \end{array}$$

In the last division 9 has been put at the end where the division really admitted only 8, 3474 being greater than 3460—but greater by 14 only, which in comparison with

the numbers themselves amounts to very little. This computation consequently agrees after all very closely with that by which the second table was made. The latter gives 1·4905. In the present computation, too, we should have found the 9, but that the secant of 80° was shortened of the 8 thousandths belonging to it.

In the triangles of the second class the perpendicular is to be regarded as at the same time radius and tangent, the former for the larger, the latter for the smaller right-angled piece, determined by the larger and the smaller angle at the perpendicular. Therefore, we may compute as in the former second example. The process, in the case of triangle Fig. 6, will be:

50	60
100, 119, 155	100, 173, 200
$100 : 119 = 173 : \frac{173 \times 119}{100}$	$1\cancel{0}\cancel{0} : 119 = 2\cancel{0}\cancel{0} : \frac{119 \times 200}{100}$
173	119
119	2
1557	238
173	
173	
205·87	
100	
305·87	
155)305·87(1·97	155)238(1·53
155	155
1508	830
1395	775
1137	550
1085	465
52	85

These numbers have all the accuracy here to be demanded. Hence, the change of computation that might

be made by means of the perpendicular *be*, which falls outside the triangle, is in this case unnecessary. With Fig. 7, however, such a change becomes more useful. It will be shown after the reckoning has been conducted in the usual way:

55	80
100, 143, 174	100, 567, 575
$100 : 143 = 567 : \frac{567 \times 143}{100}$	$100 : 143 = 575 : \frac{575 \times 143}{100}$
567	575
143	143
1701	1725
2268	2300
567	575
810·81	174)822·25(4·72
100	696
174)910·81(5·23	1262
870	1218
408	445
348	348
601	97
522	
79	

By comparing the second table, we find for this triangle in the sixteenth column, seventh row, the numbers 5·2192 and 4·7173. The computation, therefore, especially in the case of the first number, is more noticeably at fault than usual. This can be avoided.

The reader has been reminded already that in the case of the larger tangents and secants the lacking thousandths do not matter so much as in the case of the smaller. Hence, the larger are to be regarded as more accurately given. In case of choice we shall prefer them for computation.

In the present case, the better choice is possible. Look at the perpendicular *be*, in Fig. 7. It is dropped to the

production of the secant of the larger right-angled triangle from the opposite apex of the smaller. Thus originates triangle *eab*, which is a part of a larger triangle, *ebc*, with which it has the right angle at *e* in common. *ea* is the radius, *eb* the tangent, and *ab* the secant in *eab*; on the other hand, in *ecb*, *eb* is the radius, *ec* the tangent, and *bc* the secant. How shall we find the angles in the triangles? *c* has 10°; *b*, in the whole triangle *ebc*, must, together with *c*, amount to 90°, and hence by itself to 80°. Subtracting from the latter number 55°, there remains for the little triangle *eab*, at *b*, an angle of 25°. Consequently, the same triangle, in order to complete 90° for the two acute angles, has at *a* another of 65°. Now we can put down the numbers for *eab* and *ebc*. By proportion we first reduce them to the same standard; then subtract *ea* from *ec*, and at last properly divide by *ab*, which, being the smallest side in *abc*, must become 1. The advantage is that the tangents and secants of 65° and 80°, instead of as formerly of 55° and 80°, enter into the computation. The difference in correctness in the numbers here known for 65° and for 55° respectively is not too small to rectify noticeably the results previously found.

65	80
100, 214, 236	100, 567, 575
$100:214 = 567:\dfrac{567 \times 214}{100}$	$100:214 = 575:\dfrac{575 \times 214}{100}$

$$\begin{array}{r} 567 \\ 214 \\ \hline 2268 \\ 567 \\ 1134 \\ \hline 1213{\cdot}38 \\ 100\phantom{{\cdot}00} \\ \hline 1113{\cdot}38 \end{array} \qquad\qquad \begin{array}{r} 575 \\ 214 \\ \hline 2300 \\ 575 \\ 1150 \\ \hline 1230{\cdot}50 \end{array}$$

236)1113·38(4·71	236)1230·50(5·21
944	1180
1693	505
1652	472
418	330
236	236
182	94

The greater necessity for changing the computation in Fig. 7 rather than in Fig. 6 originates principally because in the latter when using proportion the numbers multiplied were not so great, and consequently did not multiply so many times the errors in the small tangent and secant. Since the large numbers obviously come of the large angles or of the more elongated form of the triangle, the more careful computation here employed is especially needed for the triangles in the last columns, with the exception of those in the uppermost rows in which the common computation employs the larger numbers anyway.

The triangles of the third and fourth classes are in the front half of the table. We have noticed already the repetitions which occur here, since the perpendicular may be dropped from any apex of the triangle, thus dividing the triangle into rectangles in three different ways. From this results the arbitrary possibility of any one of three computations; or, rather, the question which of three triangular divisions will give the safest computation.

The difference between the last two classes consists in the fact that in the third class it is still possible to consider the perpendicular as radius for one of the right-angled sections, while in the fourth class the perpendicular must be taken as the tangent in both sections. Now, from the former computations we recall that in proportion the

first number was always 100—not only an easy divisor, but admitting no uncertainty as to the correctness of the number, the 100 being a radius intended as 1, and only conceived a hundred times larger in order that we might be able to treat the decimal fractions as integers. The radius being itself the measure for the other sides, there is no question in its case as to the possibility of some lacking thousandths, tens of thousandths, and so on, whose absence in the case of tangents and secants causes always some slight uncertainty. If, on the latter account, our computations already are inexact as to thousandths, they will now become even more uncertain, since in the fourth class we are compelled to use, instead of the number 100, a tangent, the liability to error increasing as size decreases; and here it is the smallest number which from a safe changes into an unsafe one.

In order to escape this inconvenience as long as possible, the first rule for the third class is as follows: Never divide the triangle so as to make both angles at the perpendicular less than 45°, and thus the opposite sides radii, but have a care to make at least one angle of more than 45° at the perpendicular, and therefore the perpendicular itself the radius for one of the right-angled pieces. Remembering also that larger tangents are given more accurately than smaller, there will be no further doubt as to the course to be taken in computation.

In Fig. 8 the perpendicular *ce* is entirely unserviceable, for it would be tangent in both the triangle *bce* and the triangle *ace*. The remaining choice between *ad* and *bf* is decided by *bf* being the tangent of a very large angle at *a*, and having, therefore, as to *ad*, the advantage of greater exactness. The whole triangle *abc* is to be divided, then, into *abf* and *bcf*, making the angle at *a* 85°, that at *c* 40°; consequently giving to triangle *bcf*, at

b, 50°, and making the computation take the following well-known course:

50	85
100, 119, 155	100, 1143, 1147
$100:1143 = 119:\dfrac{119 \times 1143}{100}$	$100:1143 = 155:\dfrac{155 \times 1143}{100}$
1143	1143
119	155
10287	5715
1143	5715
1143	1143
1360·17	1771·65
100	
1460.17	
1147)1460·17(1·27	1147)1771·65(1·54
1147	1147
3131	6246
2294	5735
8377	5115
8029	4588
348	527

The triangle may be found in the tenth column, first row. The numbers are perfectly accurate. Hence, the same triangle, on account of the repetitions, occurs also in the tenth column, seventh row, and in the seventh column, first row. In the three different places the perpendicular divides the three different angles. At the two last-named places this division was inconvenient for computation. We had to find the triangle at the first place in order to be led at once into the right course by the indication in the table that points out the division of the angle at the apex. One should make it a general rule to look for triangles of the third class in the upper rows, but in the rear columns, where we always find one angle at the

perpendicular over 45°, and the other as small, and therefore the corresponding angle at the base and the tangent of the latter as large as is possible in the triangle.

In the case of triangles of the fourth class, on the contrary, in order to diminish inexactness as far as possible, select from the first rows and first columns, for there the angles at the base and the tangents belonging to them are greatest.

Triangle Fig. 9 occurs in the sixth column, fifth row, and again twice close by in the next column and row. It should be taken, however, at the first place. In the figure use the perpendicular *bf*, since at *b* it has the acutest angles adjoining it, those at the base being all the greater for it, having 60° and 65°.

60	65
100, 173, 200	100, 214, 236

The first 100 here stands for *cf*, the second for *af*. Both pieces of the base have become radii; but this was inevitable, as a look at the figure shows. Whether we take *ab* or *bc* as the base, either one would divide into pieces each smaller than the dividing perpendicular. As compared with the sections of its corresponding base, *bf* is at least the largest perpendicular. In one case it is the tangent of 60°; *fc* being the radius and therefore equal to 100, *bf* is equal to 173. In the other case it is the tangent of 65°, or, *af* being the radius and therefore equal to 100, *bf* is equal to 214. The second time it is measured by the smaller measure; hence, the number is larger. The other two numbers belonging to the triangle *cbf* must be enlarged at the same ratio, since all the sides are to be reduced to the same—to wit, the smallest measure. Hence, 100 and 200 increase at the ratio at which 173 increases to 214. This originates the following proportions:

$173:214 = 100:\dfrac{214 \times 100}{173}$	$173:214 = 200:\dfrac{214 \times 200}{173}$
214	214
100	200
173)21400(123·699	173)42800(247·398
173	346
410	820
346	692
640	1280
519	1211
1210	690
1038	519
1720	1710
1557	1557
1630	1530
1557	1384
73	146

The divisions are here continued in decimal fractions by means of zeros appended to the remainders, representing the remaining tenths as ten times as many hundredths, and so on. The procedure was necessary here because the resultant numbers have to be used in further computation. For this purpose they must not lack the appended tenths and hundredths. That even thousandths were found is because the thousandths in this example visibly amount to almost a hundredth. We shall for brevity take them as such, writing instead of 123·699, 123·70; and instead of 247·398, 247·40. In the first place we must add *cf* and *fa*, then divide by the smallest side, which here will be the resulting sum, *ac*.

$$\begin{array}{r} 123{\cdot}70 \\ 100 \\ \hline 223{\cdot}7 \end{array}$$

In order to divide conveniently by this number, which

still has seven tenths appended to it, think of it as ten times larger. It becomes 2237. In order to remedy the mistake thus arising, the numbers to be divided are also to be taken ten times larger. This will amount to a multiplication by 10 followed by a division by 10, which cancels the multiplication.

```
2237)2360(1·05          2237)2474(1·105
     2237                    2237
     -----                   ----
      12300                   2370
      11185                   2237
      -----                   -----
       1115                    13300
                               11185
                               -----
                                2115
```

The caution with which this computation has been conducted is rewarded by its being almost exact even in the thousandths. In the second division the last remainder is so large as to allow nearly of the writing of a 6 instead of the last 5, precisely as required by the second table, which gives for this triangle 1·0572 and 1·1064.

If the teacher will explain everything more definitely, or, rather, set forth everything more in detail, the methods indicated will make it possible for the children to compute the ratios for every triangle given in the second table. But it must not only remain possible; it must become actual. A few scattered instances would merely be examples in arithmetic. The manifoldness of the triangles themselves should be exhausted. It must become, in the truest sense, an object of knowledge. To the mathematician the possession of methods for helping himself, if a case arises, is sufficient. But for the culture of sense-perception, actual acquaintance with all the possible cases and their ready and definite discrimination are the main object. Computation is only an auxiliary for getting at the

main point. The eye is to be caused to note even slight differences in the positions of three points. It is to compare and to measure against each other the different distances of these points. It is to take heed of how much or how little a pair of lines are inclined toward each other. To speak generally, we must bring the eye to the point of shaping what it sees and of fixing what it has shaped. It must single out from the innumerable relations offered by a single view certain chief relations, and erect upon the latter, securely progressing, the edifice of the rest. But in order to this the eye must be occupied primarily with the simplest fundamental forms. These occupations must be such as to make these forms, which are intrinsically devoid of charm, objects of reflection, and, as such, important and, if possible, interesting, while changing them from immediate percepts into concepts, which can be discussed and on which common judgments can be rendered. To all these purposes computation serves, and for them it must not fail to be used.

None of the examples by which computation is elucidated and practised must be lost. The results must be entered for preservation each time. To gather up the products of diligence, even if it had but the appearance of purpose, would be advisable if only for the reason that it is neither enjoyable nor a good habit to perform labour that is lost. In this case the collecting of triangles computed for the sake of practice serves as the foundation for a larger collection, even possibly as an incitement to perfect an already half-won possession.

Therefore, even though the art of reckoning has grown sufficiently easy, let the computation of the remaining model triangles be finished. The pupil will make up for himself a table which will coincide with the second table here appended, except that it will not give as many deci-

mal places. Where several pupils are studying together the work can, to some extent, be divided. The better way, however, is to let them only verify each other's computations, while otherwise all work out every problem. Thus every one will find occasion to draw every triangle for himself, and the eye will become uniformly acquainted with them all. In addition to the numerical table there should be drawn up a corresponding table of figures. In the latter, all the triangles as far as feasible should be drawn on a variety of scales, making the shortest sides equal always, the greatly elongated triangles, however, being drawn half size. Both tables, further on, serve many uses.

VII. Episode.—Computation of the Intermediate Triangles.

For sense-perception it may suffice to distinguish the model triangles, and to be able to indicate, in the case of any given triangle, between which of them it lies. But in order to prepare for mathematics it will be to the purpose to consider also the continuity between these points—in other words, the possibilities that are passed over by the triangles in the tables.

It is plain that in the first table there may be rows between the rows, columns between the columns. Were the angles at the perpendicular to progress not from 5° to 5°, but from degree to degree, very many more triangles would enter the table. It would thus expand from within. The triangles now present would be dispersed among the others, but without the slightest disturbance in arrangement. But the space between any two would seem to have enlarged by receiving four additional triangles, and only seem so, for the distance of five degrees neither in-

creases nor diminishes by being traversed by shorter or longer steps.

If the angles should proceed by minutes only—by seconds only—the number of intervening rows and columns would be still more increased. It is absolutely indeterminable how many rows and columns there might be, for even the step from second to second is infinitely subdivisible.

Still, any triangle might be regarded as consisting of two right-angled triangles. The preceding methods of computation would remain applicable, provided one only possessed the numbers for the intervening tangents and secants.

The mathematician has for these lines printed tables. The A B C of Sense-Perception knows of nothing but its table, portable in any memory, never through life to be lost, by which we have computed hitherto. If this table were weighted with new burdens, it might break. It behooves us, therefore, to take care to put our little possession to its utmost possible use by reflection. However, in doing so we shall sensibly feel the limits, which we can not break through without science. To awaken this feeling is the chief aim now.

To begin with, objectify once more by drawings, movable staffs, etc., the accelerated increase of the tangents and secants when the angle progresses uniformly. The tangents of 46°, 47°, 48°, 49° traverse, no doubt, the difference between those of 45° and of 50°. Just so the difference from 50° to 55° is traversed by the intervening tangents for 51°, 52°, 53°, 54°. So, likewise, each of the following differences visible in Fig. 1 between the numbers 1, 2, 3, 4, 5, 6, 7, 8 will be divided into five parts by inserting between the tangents drawn in the figure all those pertaining to the angle in its progress from degree

to degree. But will the parts into which each difference is divided be five equal parts? Certainly not. The first parts will be smaller, the latter greater. Yet, if we desired to give not an exact but only an average indication by how much the tangents grow in this or that region, each of the differences might, to this end, be divided into five equal parts. Here, having no means of determining the parts accurately, we must needs content ourselves with at first making them equal, and rectifying them, perhaps, to some degree subsequently.

The following table, besides the known tangents and secants, shows their differences, and each difference is furthermore divided by five. For example, the difference for the tangents of 45° and 50° is 19 hundredths, of which the fifth part is almost 4 hundredths:

Tangent.		Difference.	÷5=	Secant.	Difference.	÷5=
45°	1			1·41		
		0·19	0·04		0·14	0·03
50°	1·19			1·55		
		0·24	0·05		0·19	0·04
55°	1·43			1·74		
		0·30	0·06		0·26	0·05
60°	1·73			2		
		0·41	0·08		0·36	0·07
65°	2·14			2·36		
		0·60	0·12		0·56	0·11
70°	2·74			2·92		
		0·99	0·20		0·94	0·19
75°	3·73			3·86		
		1·94	0·39		1·89	0·38
80°	5·67			5·75		
		5·76	1·15		5·72	1·14
85°	11·43			11·47		

On an average, then, according to this table, the tangents will grow between 45° and 50° by 4 hundredths for one degree; between 50° and 55°, by 5 hundredths; be-

tween 55° and 60°, by 6 hundredths, and so on. By this means the tangent of 46° is found to be about 1·04; that of 47°, about 1·08; that of 51°, about 1·24—that is, 1·19 plus ·05; that of 58°, about 1·61—that is, 1·43 plus thrice ·06, since 58° equals 55° plus 3°; that of 59°, about 1·67, and so forth.

Now this must be rectified so as to make the first in each of five increments smaller, and the latter, or at least the last, greater than is indicated by the average. Up to 65° the correction for the present purpose is easy. Drop simply every time one hundredth. For instance, instead of 1·04 put 1·03; instead of 1·08 put 1·07; instead of 1·67 put 1·66, etc. Thus the last increment becomes of itself greater. The tangent of 59° being 1·66, and the following of 60° being 1·73, this progress, the last one of the five between 55° and 60°, amounts obviously to ·07. It is therefore ·01 larger, and was bound to be so, since the four preceding tangents were all taken ·01 smaller than is indicated by the average.

Obviously the correction is very crude, merely made by estimate and haphazard. In what respect and to what extent it is utilizable we could have only poor notion, if the large tables of the mathematicians did not furnish us confirmation. But this correction does not hold beyond 65°. For does not even eyesight show that the larger tangents deviate from the average by very much more than one hundredth? Simply try to determine on this plan the tangents between 75° and 80°. Throw away, if you will, more than one hundredth. Use every care to divide the whole difference, here according to the table amounting to 1·94, into five unequal, ever increasing parts. Will you guess the correct numbers? They are found in the following table:

		Difference.	
75°	3·73		
		0·28	
76°	4·01		4·12
		0·32	
77°	4·33		4·51
		0·37	
78°	4·70		4·90
		0·44	
79°	5·14		5·29
		0·53	
80°	5·67		

The last numbers, which are bracketed, are those that would have resulted according to the given average—that is, which would require to be rectified. In the average, every difference would be 0·39; this example shows how the first three differences are smaller, while the last two are greater.

The preceding illustration will indicate sufficiently how the teacher ought to direct the attention and to excite inquiry by experiments on the rate of increment of magnitudes not increasing uniformly, such as tangents and secants, before giving the numbers themselves. The same suggestion ought to be followed in presenting the elements of mathematics, logarithms, sines, cosines, etc. Let the teacher add one additional observation. The progress of the tangents and secants, however lacking in uniformity, is nevertheless throughout determined necessarily and completely by the further and further opening of the angle. Hence, there must certainly be some universal rule stating in general terms this necessity, this dependence of tangents and secants on angles. Thus the teacher will produce a conception of mathematics as the science of such rules, a conception many students do not possess even after completing their entire course in so-called pure mathematics.

Below we give, from degree to degree, the tangents and secants above 65°. It is understood that they are not given to be learned by heart. They are a present from the teacher to those pupils who have learned to appreciate it. This present will be preserved in writing, and serve for preliminary practice in the use of mathematical tables.

	Tangents.	Secants.		Tangents.	Secants.
66°	2·24	2·46	78°	4·70	4·81
67°	2·35	2·56	79°	5·14	5·24
68°	2·47	2·67	80°	5·67	5·76
69°	2·60	2·79	81°	6·31	6·39
70°	2·75	2·92	82°	7·11	7·18
71°	2·90	3·07	83°	8·14	8·20
72°	3·08	3 23	84°	9·51	9·56
73°	3·27	3·42	85°	11·43	11·47
74°	3·49	3·63	86°	14·30	14·33
75°	3·73	3·86	87°	19·08	19·10
76°	4·01	4·13	88°	28·63	28·65
77°	4·33	4·44	89°	57·29	57.30

We are now prepared to compute intervening triangles. Let a triangle have the following angles: 74°, 43°, and consequently a third of 63°; the ratio of the sides is required. The triangle is acute-angled, of the third class. According to the foregoing rules, we are, for the purpose of computation, to look it out in the upper row, in the rear columns. Under the third row conceive inserted another row, which will have at the left of the base an angle of 74°. It runs through all the columns. Among others, it strikes the tenth column, where the angle at the right of the base is 40°. This angle would change into 41°, then into 42°, and then into 43°, if we set between the tenth and ninth columns intercalary columns. The angle at the apex gains what the others lose, and reversely. Here it gains 1°, and at the same time loses 3°. It consequently loses in all 2°, and from 65° becomes 63°. Thus

the place of the triangle in the first table is determined. How the angle at the apex is to be divided is indicated by the angles at the base, as it must furnish the complements to make each = 90°. The 63° divide into 16° to complement the angle of 74°, and 47° to complement the angle of 43°. We shall use, therefore, in computation the tangents and secants of 74° and of 47°. Those of 74° appear in the table just given. How to find those of 47° has also been shown. To the tangent 1 add twice ·04 less ·01. To the secant 1·41 add twice ·03 less ·01. The tangent will be 1·07, and the secant 1·46. The computation then proceeds altogether as before. Its result is that the sides have almost the ratio of 1 to 1·3, and to 1·4; or as 10 to 13 and 14. By comparing the second table, it will be seen how these numbers fall between those there given.

The following exercises a teacher familiar with logarithms can very readily increase at pleasure:

Given angles.	Ratio of sides.
17°, 93°, 70°...........	1, 3·415, 3·214
119°, 60°, 1°...........	1, 50·11, 49·62
143°, 15°, 22°...........	1, 2·325, 1·447

As regards the form of instruction, we wish once more to insert the general observation that the student, when the angles have been given out, should always make on his slate a sketch, however crude, of the approximate shape of the triangle, and keep it before him during computation. This keeps present to his mind the meaning of the numbers and prevents confusion.

VIII. Gathering the Results.—Trigonometrical Questions.

The sum total of the required elementary percepts has now been brought together. Each has its number. By this means it is not only indicated and fixed, as in lan-

guage a thought is fixed by its word, and everything by its name, but it is also comprehended as to its essence, and the concept is properly expressed. Pure form without magnitude is no object of bodily vision at all. Relations of form are reached only by numerical concepts.*

But imagination mediates between the concept and the percept. Without entirely banishing magnitude from form, it makes magnitude accidental by enlargement and by reduction. For this reason transition is rendered easy from figure to number in larger and smaller representations of an identical form, for which we introduced both the exercises in making drawings larger and smaller, and the instrument indicated in Fig. 4. If, therefore, the teacher has well administered his duty—if he has not allowed his pupils to lapse into mechanical computation—the eye, the imagination, and the understanding must by this time be equally accustomed to, and on friendly terms with, our triangular model forms.

Now, the important point is to make a good combination of all this detail, and elevate the many concepts of number into a unit of thought; to present them as transitional, as flowing into one another, and thus as constituting one continuity. For this purpose we need, in the first place, an attentive observation of the second table, and then a few exercises which give occasion to search and traverse this table in all directions.

In the second table all repetitions are omitted. Thus we now find at the left corner the equilateral triangle alone, and the farther from this corner the more we remove from equilaterality. Every column terminates in an

* Numerical concepts do not, however, attain what is properly spatial, namely, distance generally, and position or angles. For this reason an A B C of Sense-Perception must not fail to observe its peculiar difference from merely objectified arithmetic.

isosceles triangle; even above the columns commence alternately perfect or almost perfect isosceles triangles. The difference between the lower and the upper isosceles triangles consists in this: Below there is always one side equal to the smallest side, and hence equal to 1; the other side, being the smallest side itself, is omitted from the table. Above, on the contrary, two equal numbers appear, both greater than 1; they indicate the equal sides; the omitted smallest side is the base. In the lower triangles, therefore, the base is greater, in the upper triangles smaller, than the equal sides.

Precisely because going from the left to the right we remove farther and farther from equilaterality, the numbers in the rows constantly increase; they signify sides which compared with the smallest become ever greater. The angle at the right of the perpendicular* opens farther and farther, thus the right side and the base increase. On the contrary, the angle at the left remains constant throughout each row, and with it remains unaltered the smallest side. For all the triangles occurring in the second table have, it will be observed, on account of the arrangement of the table, the smallest side in every instance at the left, the intermediate at the right, the greatest lying below as the base. This arises from the largest angle, the repetitions in the first table being cut

* For angles always compare the first table. In the second, the numbers for the angles could not well be placed so as to indicate their position. On the contrary, at the right, beside the rows, those angles are indicated which conformably to the presentation here chosen must be thought of as being at the left; above are those that belong at the right. The larger numbers indicate the angles at the base, and the smaller those at the perpendicular, which, taken together, constitute the angle at the apex. Practise the imagination so as to think the numbers into their places at once.

off, being always situated in the apex, and the smallest to the right, which determines the opposite sides.

A slight confusion might arise in the use of the word base with the upper isosceles triangles. This may, however, be obviated easily by a caution; namely, of the equal sides in this case one lies below, and the smallest side, which should be the base, if the triangle had its accustomed position, is placed, as always, at the left.

A little less easy than in the rows is the explanation of the progress of the numbers in the columns. In the first place, separate the acute-angled triangles, retaining only the obtuse-angled, or, in other words, that portion which lies at the right of the diagonal running through the right-angled triangles. In each column the angle at the right of the perpendicular remains unaltered; that to the left opens, and through it increases, in addition to the base, the smallest side. Now, it is the latter with which the other sides are compared. It is their measure. When a measure becomes greater it is no longer contained as many times in that which is measured; the number indicating how many times it is contained becomes smaller. This is the reason why the numbers decrease as we descend the columns. At last the left side becomes equal to the right. This makes the triangles isosceles and terminates the column.

In the obtuse-angled triangles, for the reason suggested, both the numbers decrease. The base, to be sure, increases also as the smallest side enlarges. But let the latter become greater and greater—let it become infinite—the base then becomes also infinite. The difference between the two as compared with the sides themselves becoming less and less considerable, they may be considered almost equal. Calling, then, the side 1, the base, too, is hardly more than 1. Here in our columns, to be sure, the left

side must not exceed the right; still this observation explains to some degree that the base, though it increases, approaches nevertheless the ratio of equality with the smallest side, and hence that its number, which must express this approximation in ratio, can not become larger, but only smaller.

This latter observation does not fit the acute-angled triangles found above the diagonal running through the rectangular triangles. In them the larger number, which indicates the base, is seen to increase constantly; only the smaller number decreases. The number for the right side, of course, must decrease, as the right side remains unaltered while its measure, the left or the smallest side, increases. This will be understood from the preceding. But it has been just shown that the number for the base need not necessarily increase by reason of the base itself increasing; and now we find, none the less, that the number here increases with the line. The former is true of obtuse-angled, the latter of acute-angled triangles. But how can this distinction of triangles cause the base, which increases in both cases, to receive in the former case decreasing and in the latter case increasing numbers?

For the mathematician this difficulty does not exist; he knows from the relations of the sines that it can not be otherwise. But here the matter can not be elucidated clearly. It must be taken notice of as one of the future questions in mathematics.

To some extent Fig. 10 may be used for elucidation. Compare triangles *abc* and *aec*. When the lines *ab* and *ae*, by the further opening of the angle, merge respectively into the nearest dotted line, what consequence follows for the base and for the smallest side? Both gain; but if the angle at the perpendicular is small, as in the case of

ab, the increase of the smallest side is insignificant; the base increases much more. Hence the measured gains far more than the measure. If the angle at the perpendicular is, on the contrary, large, as in the case of *ac*, both gain equally. It is an additional point to consider whether the growth of the base amounts to much in comparison with the base itself. This depends upon its size, and, in connection with it, on the size of the angle at the other side of the perpendicular. If it extends as far as *f*, its ratio of growth does not amount to as much as if it extends only as far as *c*. Taking all this together, this much is plain: that the angle at the apex, comprehending the two at the perpendicular, must not be too large if the base is to grow more in proportion than the smallest side. If it be greater than 90°, then the second table tells us that the smallest side, in comparison with itself and in comparison with the other sides, grows more than the base, making the number for the latter, therefore, smaller.

Our considerations—and they must not fatigue the patience, as they are necessary to the use of the second table—have not yet been made exact and definite enough. It is not sufficient to know merely that certain magnitudes increase or decrease; we must also inquire how far, how rapidly, they progress. And here, especially, we must take into consideration the difference between the two numbers pertaining to one and the same triangle.

Traverse the rows. It appears that the numbers grow with ever-increasing rapidity. This will be explained at once by imagining the triangles, and remembering how the angle at the right of the perpendicular accelerates its tangent and its secant more and more the farther it opens. Finally, it is also plain that this growth is not confined to the numbers in the table, but continues to infinity if the angle is opened beyond 85°.

Traverse the columns; first the hindmost. Behind the last in the table there would be columns were the rows lengthened. These columns would commence above by far greater numbers. Even the hindmost column in the table has incomparably greater numbers than all the other columns. It terminates, however, in 1 and 1·9924. It traverses, therefore, the identical numbers which also occur in the other columns. This is true of every rear column in relation to those preceding it. This circumstance makes it somewhat troublesome to assign to given numbers their place in the table. Numbers not amounting to much over 1 and 2 might singly be brought in almost anywhere. But for any determinate triangle there are always two numbers given. The point is to find the place into which both will fit simultaneously. If, for example, 1·6 and 2 are given: the number 1·6 appears at several places in the table—for instance, in row thirteen, column fifteen; but we do not, at the same time, find 2 but 2·4; therefore this can not be the place for the numbers. To find the place one must have got one's bearings among the differences of the numbers which belong to the same kind of triangle.

For this purpose we assume certain points of view in the table whence we may survey it.

Traverse the diagonal of right-angled triangles from right to left, omitting alternate triangles. Between the numbers 11·43 and 11·47 we find but little difference; between 3·73 and 3·86 it is somewhat more than 1 tenth; between 2·14 and 2·36 a little over 2 tenths; between 1·42 and 1·74 a little over 3 tenths; and between 1 and 1·41 somewhat more than 4 tenths.

Traverse row nine. It divides the field of obtuse-angled triangles through the middle. Here we find the difference to be between 1·41 and 1·93, about 5 tenths; between 2·73

and 3·34, about 6 tenths; between 8·113 and 8·789 it is not quite 7 tenths.

These differences should be remembered.

At the end of each column the difference of the numbers belonging together is visible at once. It amounts to exactly the decimal of the lower number, the units cancelling by subtraction.

Near the head of the columns there is no or almost no difference, but it grows continually till it attains to the decimal fraction just mentioned. The differences which we have but just noted serve in a degree to trace this growth, because most columns—and these the larger—are cut by that diagonal and by that row.

We are now sufficiently prepared for the solution of the following questions which belong to trigonometry:

It may be required to make a triangle out of three sides of given lengths; or the sides of a triangle may be known, the angles unknown. In this case the problem is to find the angles. Similarly, two sides and one angle may be given. In this case, the third side and the remaining two angles are to be found.

In these problems we must separate form from size.

If, for example, the lengths of the sides are 2, 3, and 4 feet, the triangle will certainly have a definite size. This the second table ignores. The smallest side in it is always 1. But the 3 and 4 feet may also be measured by the 2 feet—i. e., the inquiry may be how many times these are contained in those. The measurement is performed by dividing 3 and 4 by 2. If the division is continued into decimals, the resulting numbers must either occur in the second table or it must at least be possible to assign their places among its numbers, since all the numbers in this table signify nothing else than how many times the smallest side of a triangle of altogether arbitrary

magnitude is contained in the other two sides. To find that place is the kind of solution of these questions which is suitable to the A B C of Sense-Perception. Methods resembling those of mathematics would be as devoid of purpose as they would be impossible. Here the triangles are considered as objects of knowledge, not of computation. Our sole concern is to recognise them by the ratios of their sides as well as by their angles, and to find them out among the other triangles. The division required is here very easy:

2)3·0(1·5	2)4(2
2	4
1·0	
10	

We must find, then, in the table the numbers 1·5 and 2. That we shall not be liable to mistake the 2 in row six, column twelve, for our 2, is self-evident, since in that place the number which belongs with the 2 is 1·7. We have to look in some place in which the difference can amount to 5 tenths. We have therefore wholly missed the region, though close to that 2, at the left, occurs a number, 1·5098, that seems to coincide with our 1·5. On the contrary, we must seek for orientation in row eleven, at the numbers 1·41 and 1·93, which have the required difference. From this point in what direction shall we turn? The numbers must increase; certainly not, therefore, to the left. Nor straight up or down; the difference would be too large in this direction, too small in that. But straight on in the row the difference increases. Diagonally down it grows likewise. Hence, nothing remains but to mount very slightly diagonally to the right. It is obvious that here the numbers 1·41 and 1·93, while increasing into those neighbouring them, must pass through

1·5 and 2. Hence the triangle is to be located between rows thirteen and nine, and between columns twelve and thirteen. It has therefore one angle between 50° and 45° and another between 20° and 25°.

The finding of the direction whither we must turn was made a little difficult intentionally for the sake of practice in surveying the table. Merely by comparing row eleven with the isosceles triangles the direction is apparent in which approximately equal differences are to be expected. The decimal fraction ·5321 below, in column ten, taken together with the numbers 1·41 and 1·93, or the decimal fraction ·7320, together with the last numbers of row eleven, whose difference also amounts to nearly 7 tenths, indicate this direction, only the latter indicate it somewhat too obliquely. It is indicated, however, with especial distinctness by the upper isosceles triangles, when taken together in a line. In them the difference is no doubt equal. It is 0—that is to say, there is none.

With the aid of our last remark it is no longer difficult to assign to all the triangles determined by the sides their positions in the table, and thus to find their angles. Let the sides be 3, 4, and 5 feet long; 4 and 5 are first divided by 3:

3)4(1·33	3)5(1·66
3	3
10	20
9	18
10	20
9	18

Difference: ·33. . . . To find this, follow the indicated diagonal direction, starting from the middle between the lower ends of the columns eight and nine. The diagonal of right-angled triangles is soon struck. A triangle of the

given sides actually is right-angled. Its numbers, it is apparent, fall midway between rows seven and eight in the transition from column ten to column eleven. Besides the right angle, it therefore has another between 50° and 55°.

Let the sides be 3, 8, and 9 feet. $9 \div 3 = 3$ and $8 \div 3 = 2{\cdot}666\ldots$ The difference, as before, is ·333. . . . Continue in the former direction, toward the right. The numbers 2·53 and 2·87 still give the difference too large, and are themselves too small. A little farther upward we come to numbers that are too large already. The triangle, therefore, lies between rows five and six and columns fourteen and fifteen. It has angles between 65° and 60° and between 20° and 15°.

Let the sides be 10, 13, 14 miles—whether miles or feet makes here no difference. The numbers will be 1, 1·3, 1·4; the difference 0·1. This difference will be found close under the line of the upper isosceles triangles. The numbers fall between rows five and six, and columns nine and ten. The angles are between 55° and 60°, and between 45° and 40°.

Let the numbers be 10, 19, 25, or 1, 1·9, 2·5. The difference will be 0·6. A little above the nethermost numbers in column eleven begin to go upward in the well-known direction. We come to the numbers 2·06 and 2·64, which are too great already. The triangle lies between rows nine and ten and columns thirteen and fourteen, and has angles of nearly 45° and 20°.

Let the numbers be 1, 1·8, 2·6—difference, 0·8. Angles very nearly 30° and 15°.

By the aid of the instrument Fig. 4, it is very easy to practise this work.

When the problem is solved, let the triangle be sketched each time.

It is obvious we have here the regress from given concepts to the corresponding percepts, as formerly, in computing the sides from given angles, progress occurred from the percepts to the concepts.

If it be desired to connect the percept and the concept still more immediately, still more simultaneously, the second problem mentioned above may be used, which gives two sides* and one angle. It thus gives the triangle partly by concept, partly by perception, but by both only imperfectly, the student being left to find the complete determination. At any rate, this problem could be solved from the table or by computations alone only with difficulty. Being now entitled to suppose practice in drawing and an acquaintance, already somewhat familiar, with the second table, we may adopt the following method:

A triangle—one angle of which is given—can always be drawn pretty accurately. This being done, its very shape will permit us to recognise, in a measure, in what region of the second table it belongs. We may get additional aid from the given angle, which in this particular region of the table can, of course, occur in only one line—column, row, or diagonal. Compare also the ratios of the given sides with the numbers in the table. From this the true place of the triangle may be determined with considerable exactness. The exercise requiring drawing, examples can not well here be given. It is intended only for the abler among the pupils, and for this reason we may be the rather permitted to abstain from further exposition.

Perhaps it will seem desirable that the determinations

* Do not mistake given sides for given percepts—at all events, not in respect to shape; and it is with the latter, not with magnitude, that we have to do. The sides give us nothing but their ratio, and that is a concept.

of the triangles by the sides be less uncertain. It is a matter of rendering the table more complete, and the fact is, that a rather easy method might be given for interpolating intermediate members. If it were permitted to expand the plan of the A B C of Sense-Perception beyond the necessary relations—if, for example, we desired also to practise the imagination in the presentation of physical spaces—the present elementary exercises might fittingly be increased to twice the number; but considering the mean opinion usually entertained by pedagogues of the abilities of children, even what is given thus far may perhaps appear to many too difficult and too long. At all events, to break off here is more modest.

Third Section.

USE OF THE A B C OF SENSE-PERCEPTION.

If the reader has had the kindness to follow the writer attentively to the end of the preceding section, and has not decided as yet for rejection, he will not be willing to grant to the author permission to retire absolutely at this point. Aside from the additions perhaps suitable, or possibly even essential, to that circle of preliminary exercises, it remains to fit, in a suitable manner, this circle into the other parts of instruction. Its use must become visible, and it must be definitely pointed out by what procedure the same may be realized. Otherwise this use will appear altogether chimerical, which would place our proposed exercises in the objectionable catalogue of foolish and airy projects. There is much instruction and reading which without either preparation or sequel is thrust in haphazard among other studies, and which leaves scarcely even the joyless recollection of a vain endeavour. These preliminary exercises would be nothing better if, without being

worked up any further, this raw material were now left to itself.

How little these meagre fundamental forms, by themselves alone, would effect for the improvement of sense-perception is easy to conceive. True, as early as the beginning of the First Section their occurrence as component parts in every shape, howsoever composite, and the facilitation of the articulation of forms by their means, were mentioned. In fact, the application of the A B C of Sense-Perception was there indicated as far as the general idea is concerned. Still, nothing was indicated but the concept, and this only as one member in a series of thoughts which in that inquiry served to find the materials for the A B C of Sense-Perception. But when a conception is to be actualized, when thus as a force it is to enter into combination and conflict with other forces, there arises at once the question, How can we assure to this force the measure of strength needful for it? An additional question is, Into what combination can it be put with other forces, and how can this combination be introduced into still further combinations? How far should we spin out this particular line of our endeavours? Where may we cut it off, as one principal thread ready now for the whole woof?

Sense-perception, this indispensable, this firmest, broadest bridge between man and Nature, certainly deserves, as far as it is capable of being cultivated by any art, to have dedicated to it one chief line of pedagogical endeavour. Where this line begins has been shown, as also its further direction. Except for closer reflection it possibly would now fall into the hands of the drawing teacher, who might use the model triangles in order to secure exactness in copies by causing to be noted in the originals the position of certain chief points, which would be a little neater and more convenient than the use of disfiguring nets, parallel

lines, and the like. But an advantage so small would hardly be worth the expenditure of time and trouble which the elementary foundations have cost. And, furthermore, how far we should thus be from reaching that great idea, the cultivation of sense-perception! This has for its object Nature itself. How far down any practice in drawing must be subordinated here! It would be the highest pride of the drawing teacher as well as the teacher who is cultivating the sense-perception if they could unite in bringing out by their training the desired accuracy and facility in the apperception of Nature.

But they can not join hands immediately. The leap would be enormous, from the simple triangles of the one to the extremely composite combinations of these triangles which the other would demand. Besides, the artist would tolerate them only as first auxiliaries, as the fundamental beginnings of formation. He would require the attention, though beginning with them, to turn away thence to actual contours, to the flowing curves, to what is beautiful, forgetting in it the sharp corners and angles. But the pupil, his attention being still required by the combinations of the triangles, would cling to them, unless he had acquired the ability previously to cast all this behind him as something absolutely completed and done with.

It is plain, then, that between simple triangles and the composite forms which art and Nature offer to the eye some transition must be made.

One might, for example, by adding and gradually employing in all its combinations a fourth point with three given ones, form a series of quadrilateral model forms, going thence to five-sided, to six-sided figures, intending to familiarize them analogously as formerly with the three-sided, through drawing and calculations. But is it not at once apparent in what an interminable labyrinth we should

become involved, and what a tiresome prolongation would thence be given to the preliminary exercises?

Were all these details needful, the whole enterprise would stand in danger, though its usefulness were fully conceded, of being declared practically impossible on account of the cost.

But there is no longer need of the slightest expenditure of time and pains; on the contrary, time-saving is to be hoped for.

Among the indispensable and generally introduced studies of boyhood there is one which, we might say, has waited for these model triangles, one that without them can not fulfil its purposes, and, conversely, which renders to them the perfectly reciprocal services of presenting to the eye all their combinations, more or less composite, already enlarged or reduced, and of requiring their reproduction by the imagination, and of thus rendering them familiar to both.

Geography—what is its intent? The location of certain names of provinces and cities by means of other names of countries and divisions of the earth? What, after all, is the purpose of maps? Doubtless it is to give us some sort of a—no matter how confused—picture of the relative positions of these things. But does the map fulfil its purpose if the picture remains confused? What is the intention of Gaspari's maps, which give the cities without names? What is the intention of the pedagogues who make boys copy whole maps in outline and colour? True, it is not the whole map that ought to impress itself upon the imagination uniformly, even if by such trifling it could. But, in the ratio of their greater or less importance, separate points, such as cities, capes, sources and mouths of rivers; less so, the ever-variable boundaries of countries and provinces, should be present to the

imagination in their relative position as definitely as possible. One ought to be able to traverse in thought a whole division of the earth with quickness and certainty. But this imaginary journey hastens from capital to capital, from one port to another. Minutiæ should not detain it on the way, except as here and there it may find a special incitement to sojourn. Hence, what is of minor importance must be thought of as merely interjacent, as contained in a region previously determined by more noteworthy points. The latter must be made prominent, must be separated from the rest, and must be apperceived only as connected among themselves, howsoever far apart. Immediately and without gradually creeping through what intervenes, one must be able, as to position, to picture them to the mind definitely.

This leads us straightway to triangles. It definitely presupposes the preliminary exercises; for neither more nor less than three points are in a simple, immediate relation as to mutual position. The question is now, Is the pupil able separately to fix any three points on the map, and to discriminate their position from every other possible position? Has the teacher a means for investigating how well or ill this fixing, this discriminating, has been accomplished? Can the teacher and pupil mutually communicate the apperception and test and correct it? Are both sufficiently versed in orientation, in the whole possible multiplicity of triangular forms definitely to assign to the occurring triangle its place in that wide field? For this, and nothing else, is meant by the required discrimination.

Let the teacher commence the presentation of every new map by naming and pointing out the three most important places on it. They will form a triangle. This will fall into one of the four classes which we distinguished in the Second Section, Number VI. Into which?

Let that be the first question. Next, the student should point out between which columns and rows in the table it should be intercalated. To do so, inexpert students may employ the aid of the instrument, Fig. 4, and the table of diagrams which they have previously drawn. Finally, by comparison with the scale on the map, let the eye estimate the amount of the smallest side of the triangle in miles. The two other sides result from a simple multiplication of the number of miles just found by those numbers in the second table which belong to this triangle. (Of course, in this work no great accuracy is exacted.) If even this much be difficult, let a single triangle suffice for this map. But geography is continued long enough, and hence proficiency will increase. As rapidly as it does so, take in addition to the three points a fourth noteworthy point in the map, selecting at least one of the three new triangles thus originating for similar treatment. Later on, a fifth, a sixth, and even more points may be added. The four-, five-, and many-sided figures thus originating may be compared for similarity or dissimilarity. Thus an important point may be made clear—the connection between different maps. The web of triangles may be connected with the determination of latitude and longitude. Not only will all this not retard the progress of geographical study, but its success will be considerably quickened.*

Let the bearing of the teacher toward his pupils in this application of the A B C of Sense-Perception to geography be very easy and gentle. Rapid zeal is rather for

* I should think that no one, for the sake of Pestalozzi's method, first given in his book How Gertrude taught her Children, will abandon the excellent method of Gaspari. But with the latter the advantages furnished by the A B C of Sense-Perception can be very easily combined.

Numbers III, V, and VI of the Second Section, where it is of importance boldly, dexterously, and insistently, now by encouraging and again by the pressure of authority, to lead the student over difficulties in the needful preliminaries of knowledge. At the present stage it is essential rather to conciliate favour for the triangles in order early to call forth unconstrained and spontaneous applications. Besides, in geography, being already known, they are the easiest part. For this reason they furnish by themselves welcome resting points. The tiring pupil will seek for them, and will like to rest at them.

The more happily the youthful fancy has recognised in maps the representations of the earth's surface, the more easily and unconstrainedly can our triangles mount next to the starry sky. Its brilliant points are still more obvious than the cities on the map—better suited, therefore, for apperception by the aid of our preliminary exercises, and only by their aid capable of unconfused apperception. Now let guidance change completely into companionship. Occasionally the names of the constellations may be given. Still, the pupil should not be forbidden to draw his own animals, plans, and maps in the sky.

There can hardly be a doubt possible that by these applications the eye will be rendered sufficiently familiar with the combinations of the triangles. The eye, resting upon a map or upon the sky, is unable almost at the mere thought of triangles to refrain from seeing before it a large number, since all that is required is to abolish the uniformity of vision by picking out some few from among all the points presented. The arbitrary play among triangles, quadrangles, pentagons, turning and twisting, combining and separating, building and destroying, the deal-

ing this way and that with the given material, are really the business of childhood. It is the natural course which children's observations always take. Therefore we may say, all that is necessary in the A B C of Sense-Perception is to render familiar to children the general concept of the triangle, to guide them next to the possible differences in triangles, and finally to place before their eyes objects in which occur, and are emphasized, not regularly formed lines and surfaces, but in the main only scattered points. Out of these they create triangles for themselves, together with all the possible combinations. Therefore, putting the worst case for the preliminary exercises—a case which every good teacher can and ought to prevent—namely, the case of one half of the children in a school instead of drawing and reckoning, sitting thoughtlessly, merely gaping or staring at the horn-leaves or at the instrument used in presenting the triangles. Even this mere gaping, which in this case consists at least in taking a view, will here, as perhaps in no other branch of instruction, help to attain the object to some extent. If geography follows, then, only assuming a similar act of looking at the map, the mere recollection of the triangles will awaken the eye automatically, dissolve and split up the chaos of the commingled signs for cities, and secure some degree of attention for an instruction aiding in this decomposition.

Not until now have we reached the actual limits of the A B C of Sense-Perception. It was not concluded till everything was prepared for an immediate transition to the multiplicity of natural objects; only the ending was advantageously interwoven with another instruction so as to avoid separate exertion.

To take care of the transition itself should now be the office of the drawing master. Let him not decline it, for

it will give greater dignity to his specialty; he will even cultivate it more felicitously by making his art the means for the cultivation of the apperception of Nature. Whether, for the purpose, our A B C of Sense-Perception is serviceable to him has partly been discussed in the beginning of the First Section; in part, the following proposals perhaps will tend to determine more definitely the mode of execution. First, a few preliminary observations.

The forms that Nature presents are by the eye apperceived in one way, by the imagination in another. The eye sees them flat. The imagination endeavours to represent them as they are actually extended in physical space. This endeavour succeeding partly, man falls into a vacillating, midway condition. He sees a definitely raised relief. Both art and science seek to extricate him from this condition. Drawing and perspective teach imagination to go back and to restore the visual plane destroyed by it. Taking the term *anatomy* in its broadest sense (by which miners may be termed anatomists of the earth's crust and astronomers anatomists of the sky) all species of anatomy and solid geometry, contrariwise, urge the imagination on to complete its course. They practise it in expanding the vacillating relief till the true boundaries of the body are reached. This gives us the ability to think unconfusedly, in the next place, its internal structure, the strata, lodes, and veins it contains, in their order and combination, and finally, even the changes in this order, in the case, for instance, of motions in the internal structure.

The latter business of the imagination, however, as difficult as it is important, is based entirely upon sense-perception. The imagination makes up the picture of the body from parts apperceived through sense-perception. For this reason the culture of sense-perception is so necessary a preparation for all those anatomists—that is, it is

a necessity to physicians, surgeons, mechanicians, architects, carpenters, physicists, geologists, astronomers, and, in general, to all people to whom accurate presentations of physical objects are important. This preparation is all the more necessary because, in actually learning the arts and sciences which those names indicate, the operation of the imagination is that which is constantly presupposed, not that to which attention is directed. Hence, the imperfections, the errors, that creep into this operation lurk deep, the teacher not comprehending wherein the pupil is deficient, and rejecting him, perhaps, as brainless, simply on account of clumsiness in the imagining of physical images.

This observation is placed here partly to open up a prospect into the distance through which we desire the A B C of Sense-Perception to extend its effects, partly in order to make felt the necessity of a systematic connection which ought to join in its bonds the teachers of different and in other respects heterogeneous branches, if the root of so many abilities—to wit, the culture of sense-perception—is to be properly nurtured in the mind of the pupil. We have been obliged already to call in the geographer. Nor are we able to dispense with the draughtsman, precisely because practice in seeing what is convex as if it were flat constitutes one principal branch of the trunk of sense-perception; not to mention that its apprenticeship, the apperception of surfaces, must be complete before physical images can be composed correctly and readily. In respect to the latter it might be asserted that really no whole constituted in physical space is imagined completely, unless the imagination knows how to present to itself also with the greatest facility every perspective projection of the same. However, to carry this into detail would be too prolix here.

Now, the teacher of drawing might, somewhat like the teacher of geography, begin by emphasizing a few prominent points in the engraving used as a copy by first determining their position through triangles, and then by causing the student to join to them, by the aid of other triangles, the interjacent or surrounding parts. By doing so, he would take the A B C of Sense-Perception into his service; but he would not by his service further our purpose. Whether his pupil learn to copy an engraving is of no interest whatever to us. We rather wish, for the teacher's sake, he might not need at all so dreary an auxiliary for facilitating instruction. We rather wish he might be able to avoid altogether the dubious question whether the pupil really recognises and understands the engraving as a portrait and representation of real Nature, which we hope is the master's intention. We rather wish that he had means enabling him to lead the pupil from the first successfully to draw from Nature. We, too, should thus reach our end, because what the teacher would require would be exactly this—to draw that which is convex with correct perspective upon a flat surface, and then to shade this surface so as to make it appear to change back again into the physical space which Nature herself occupies.

It perhaps needs but a slight device to help the drawing to attain perspective correctness. Our only presupposition is that the teacher will join his work to the A B C. In order to do so, he can not well make the beginning on pictures of organized beings. Everything is too round, too soft. It is not easy to pick up firm points in them upon which teacher and students can with certainty reach a mutual understanding. In landscapes, on the contrary, there occurs much that is pointed and angular, and everything is scattered more accidentally. We are reminded as yet of the map with its triangles.

In order that the prominent points in the landscape may be co-ordinated into plain triangles, it is only necessary to have these points fall into a few lines which hover before the landscape, and are in a plane which is vertical to the axis of the eye. That, as we take it, can be arranged for. Have, for example, a staff with a groove in it, in which the end of another staff at right angles with the former can be shoved up and down. Take this extremely simple machine into the open air. By a plumb-line fix the first staff, sharpened below, vertically in the ground, so as to let the student, standing before it at some little distance, see in the landscape a couple of principal points exactly on the edge of the staff. Shove the other staff till it touches some third point. Cause the student to note the originating triangle. Let us hope it will be no longer necessary to talk any more of rows and columns. Neither is there need of at once drawing the triangle on paper; in case of necessity we might at most allow the student to indicate the terminal points, but never to spoil the appearance of the drawing by the sides of the triangle. In a way similar to the above, let the entire landscape be scanned till the pupil has confidence enough to indicate without auxiliaries the principal points or lines in a sketch which must be the work of a few minutes. Let the teacher correct the sketch on the spot, and let this terminate for the time the entire exercise; only many similar exercises must succeed each other at short intervals—at least there should be one daily. The only point of importance is to learn how to see and to comprehend completely—to prove by one's own deed, the possibility of a surface representation of Nature. This work must be carried on with swift earnestness and without any foreign admixture.

Perhaps the pupil feels it to be sufficient to move the

eye hither and thither for a while before a staff, perhaps only before a tree, all the points of the landscape thus coming successively to the edge of the staff, and therefore into one plane, only this plane is not seen simultaneously; no triangle can actually present itself.

After thus achieving measurement of free Nature by the eye, neither teacher nor pupil will be any longer tied to the landscape. Attempts may be founded for winning from the plaster cast or the marble the contours of man, no longer now immediately upon the A B C of Sense-Perception, but upon the eye's perspective measurement attained by its aid. This power is no longer, so to speak, compelled to isolation on tripods. Let the teacher make the beginning. In the presence of the students let him draw from the bust. The copy thus produced certainly will represent the original; surface and body will interpret each other reciprocally. The student will grasp their mutual relations and easily try a contour himself. If the eye be well instructed, the hand will soon be compelled into obedience.

It is needless to add much. The true artist takes care without being reminded that with impressive sense-perception, which here alone occupies us, the æsthetic perception be united early. He will cover up now, so to speak, the hard triangles. He will incite the student by the gentlest curvatures, by the softest gliding of bend into bend, to conceal from his eye the model forms still hovering before the imagination, as if burying them in intentional oblivion. Thus they become, what they ought to be, the well-draped and ever firmly holding, sustaining skeleton of all drawing. Young people of talent the artist will carry far enough to enable them, after so many exercises in impressive sense-perception, to single out from the observed motion of an animal or a human being the

most beautiful moment, the most advantageous position, and to commit it for preservation to paper.

This is the place to mention an episodic contribution which another branch of instruction at the time of the drawing exercises either has yielded us already or is to yield soon. We speak of mineralogy, to the extent that it judges specimens by their external marks. And there is hardly another equally favourable opportunity for sharpening the eye even for the minutest differences of texture, glitter, and colour, and combining with these simultaneously so many other sensuous percepts. The A B C of Sense-Perception, however, need not ask here for special consideration; the unity of results will follow of itself in the student.

According to the observation made above, the consideration of imagining physical spaces should be taken up in the next place. Beams, and carpenter's work in buildings of every kind, because representing rectangular forms, would here do similar service to that which was rendered by the map to the apperception of surface. We should thence go on to machines, showing them first at rest, then in motion. Spherical forms should become familiar in continuing the contemplation of the starry heavens, which, in fact, only by their aid become accessible to a more extended survey; for the plane triangles serve only for smaller portions of the heavens, which the eye may consider flat. Only now would the preliminary notions in mathematical geography, usually placed at the beginning of this science, gain perfect distinctness; it is but too well known how deficient children as a rule are in their conception of them. Impressive sense-perception would conclude its studies with natural history. It would most

especially study the skeletons. In the interior cavities of the latter, it would consider the place for the many kinds of organs which are arranged within them, not only in a general way as to place, but as a geometrical body determined thus and so. It would compare skeletons with living beings placed by their side, in order to make clear to itself the covering of the bones by flesh and muscles as accurately as either sculptor or painter need. In skeletons of several animal species it would know how to observe the various modifications of one and the same general animal structure, not merely as a difference, but as a difference of such a kind and magnitude. It is well known that natural science, as long as it distinguishes its objects by only external marks, represents them as only accidental phenomena. Not till it traces organization and its purpose does it represent the plants as plants, animals as animals; and not until it refers whole genera and classes to the common fundamental idea, as different expressions of the latter, does it, in fact, represent Nature as Nature—that is to say, as bringing forth according to concepts. But it is equally certain that in order so to understand Nature, nay, in order even to be inclined to devote the time and attention necessary to this understanding, we must be given, in the first place by mere sense-perception, a far more definite knowledge of the immediate data of Nature, and a far closer habituation of man to Nature than can ever be gained by a superficial showing and viewing of all species of engravings and natural objects. Here we are shown again the necessity of the teacher's having some means of compelling perception—at least, in emergencies, to sharp attention. He must be able to require from it certain definite data and to bring home to it its mistakes. However, in the case of pupils who had taken a course in all the preliminary exercises, supposing an instruction not

entirely devoid of taste, such an emergency would rarely occur. On the other hand, the contemplation of external and internal animal forms, which deviate so far from mathematical regularity, would concededly be endless, but that now, not awaiting the call from the teacher, the trained analytical attention will become immersed in the view down to its elementary forms and, re-emerging, concentrate all the power and riches of its acquired simpler apperceptions into the wealth of the total percept.

After so gradual a progress through so wide a circle of manifold exercises, of which each, even when taken singly, has a specific value of its own, one may well hope to have wedded facility to accuracy. To believe that the eye through our exercises would acquire a schoolboy's hesitancy, an anxious uncertainty, would be all the more an idle fear, as the daily common use of the eye continues, of course, and accepts of the culture conferred by our art only that which is easeful and helpful to it. Of course, the children must be kept cheerful and alert in order that they may not exercise their sight in school alone. That which is seen by them with free enjoyment must be infinitely more than that which is thrust upon them by the horn-leaves and the wooden triangle. But these laws of every tolerable education are simply matters of course. Any instruction, even the most excellent, becomes deleterious as soon as the children's physical forces are not kept at an equilibrium.

We may also flatter ourselves that the circle possesses the needful completeness. None, it should seem, need fear meeting difficulties as yet unknown to him in any use of his eyes, or of their representative, the imagination, who has already taken his bearings as to machinery, as to the heavens, and as to the interior of animals. On the contrary, such a one would be conscious not only of a

powerful apperceptive faculty, but, with the least inherent mobility of mind, of an extremely useful faculty of mechanical invention also. Far from becoming confused in the schools of the geometrician, architect, physiologist, or any workshop of high or low degree, by the multiplicity of objects, he would, on the contrary, know how to acquire and continue by self-activity all the manifold instructions there pressing in upon him. He would know how to take hold with head and hand, if in the earlier years some care be taken for manual dexterity—a care which surely need not be commend specially to such teachers as are able to take an interest in the A B C of Sense-Perception.

And now this thread of instruction might be laid aside as completed; it might be left to general pedagogy to ordain the rest concerning it and its interweaving with the whole, if only there were not left in the elementary exercises a gap which robs of its necessary basis everything that is connected with the imagining of physical spaces. There is needed a preparation for the imagining of geometrical solids, similar to the preparation which the plane model triangles furnish; primarily, to be sure, for sense-perception in general; secondarily, however, especially and exclusively for superficial perception. This imagining of physical spaces requires its own model forms. It would not be difficult to fill the gap. For the present it is left purposely. Why should one erect a large edifice before it is known whether any one desire to dwell in it?

The question is, whether men of intellectual eminence are inclined to take up the pedagogical concerns here discussed, and whether the present treatment may obtain their assent.

The question certainly is not, whether the external phenomena, the first to nurture, the most faithful to attend, the most tireless to teach the youthful mind, are

deserving of having their most intimate acquaintance sought after.

The question certainly is not, whether it be good that man become truly at home and domesticated in the world of his senses, which for the present is his home and dwelling place, or whether there be happiness in being able to throw one's self into the arms of Nature with comfortable ease in any kind of business, and of missing naught at least of the sensuous clearness and distinctness, in spite of the riddles which she gives to the understanding.

Neither, it should seem, can there be any question whether there is a connection between clear sense-perception and sound judgment, between precise vision and precise thinking, or whether by a knowledge of Nature a bright mind is well prepared for occupying itself with abstract concepts, and the more sluggish man is guided by the incitement to the use of his senses into the nearest and right groove for his activity.

Can we question whether education comes up to its idea, when that which accident and ordinary instruction shed upon youth unordered is moved by education close together into a series as long as may be, progressing from member to member, as from means to end? When it gives to the materials furnished for the instruction of youth by the works of Nature and of art, by the surface of the heavens and of the earth, such a position as to practise the intuition by a continued progress in the easier forms for apperceiving the more difficult and the more composite?

In the confidence that no man of understanding will doubt all this, we have said of it little or nothing. We commenced with that concerning which doubt might arise.

Why cultivate sense-perception? Does not the eye see of itself? For the apperception of Nature, is not it by

Nature good enough? Whence the means for the culture of the imagination? Why triangles? What is the reward for their computation? For such early mathematical labours? What transition from triangles to the world?—which does not look as if it were a heap of triangles. Where is the place among and what is the connection with the other studies?

It would be unbecoming to repeat what we have said on these and similar questions. Not so, perhaps, to ask an examination and to express a desire for practical trials made with that sagacity which in adaptation to given circumstances aptly alters the inessential without displacing essential elements. For example: The right age for this instruction is determined by ability and necessity. To a good memory there might be trusted for the lattermost computations, besides the tangents, sines also. However, sines must not be taken from the beginning instead of tangents, in order to avoid injury to objectification. Slow students must reckon with only two, the most skilled might reckon with four or five figures, which may be taken from the second table. At first, one teacher will have to take care of a variety, which strictly should be divided among different teachers, etc.

What definite modifications should the mechanism of this instruction receive in order to be suitable for many students simultaneously—for schools? Should it be noticeably altered for girls? What advantages, what applications, and what difficulties will result in connection with other instructions? Above all, what involuntary effect will be produced in the youthful mind by these preliminary exercises over and above the chief effect which we hope we have foreseen rightly?

All these many questions are committed to the judgment and to the experience of others.

The A B C of Sense-Perception thus sent into the world is as yet a poor stranger who must on his honest face beg many a good gift from many hands. It might at once somewhat more copiously have been set forth. But it must first appear to deserve this. If so, that which has been retained can be sent after it.

PART III.

CONCLUSION.

We have endeavoured, through a graded series of introductory writings from Herbart's own pen, to lead by a course which might not itself seem wholly unpedagogical to a proper appreciation of the only treatise on the methodology of a specific branch of instruction which the master has bequeathed to us. The test of the A B C of Sense-Perception in somewhat extensive application, it may be permitted to add, has neither been omitted nor proved unsatisfactory. Details would, of course, be out of place, and justly open to the charge of egotism.

We have thus far emphasized the fact that, strictly speaking, the only science of education is the pedagogy of educative instruction. This rests, of course, upon psychology. We must not, therefore, close without equal emphasis upon the fact that as yet we know very little of pedagogy, because of the limitations entailed by this dependence. This is the other half of the truth. He who would understand the sphere of pedagogical endeavours must realize that it has two hemispheres.

In 1812 Herbart published in the Königsberg Magazine for Philosophy, Theology, etc., his Psychological Research into the Strength of a Representation conceived as a Function of its Duration. The problem is worked out by the liberal use of the integral calculus distinctive of Herbart. This paper is followed immediately by the essay:

On the Dark Side of Pedagogy.

Immediately after the preceding psychological essay, it is unnecessary, I presume, to remind the reader which side of pedagogy is here by emphasis called the dark side. How immeasurable a distance from the inquiry concerning the strength of a simple sense-representation considered as a function of the duration of that representation to a complete psychological theory of ethical education! And all along the road primeval night prevails, and all this road runs along the dark side of pedagogy.

In order to remain somewhat in touch with the preceding researches, let us first reflect what significance they may have for pedagogy. They furnish a fragment of a theory of attention, and a theory of attention would constitute an essential though but a small part of a psychological pedagogy.

Since on the dark side of anything nothing can be said except to the extent that points of light shine forth out of the darkness, it may perhaps be proper to look once more at the points of light just found, and to compare them with the desideratum of a more extended pedagogic knowledge.

I take for granted it has escaped no one how almost impossible it is to avoid making incessant claims on the attention of the pupils, and how harmful are some of the means—such as rewards, competitive excitement, etc.—occasionally devised for obtaining, after all, faithless attention, and how much depends on utilizing without harmful means and to the best advantage the attention that is possible from a pupil.

It is equally well known, I presume, that in being and especially in becoming attentive we feel largely passive, while, nevertheless, though in very different degrees for

different individuals, our own will controls the attention.

From the preceding investigation it is apparent how that which seems to us passive in attention—for the soul is never strictly passive—is jointly determined by the strength of the impression, the freshness of receptivity, the degree of contrast to already existent impressions, and the degree of unrest of a mind more or less occupied antecedently. From the same investigation it is apparent, not, indeed, what constitutes the activity of a more highly educated mind in controlling its own attention, but at what points the activity must be applied in order to produce the intended effect. The art in purposed attention consists in properly directing the physiological receptivity, in seeking the stronger impression, but, above all, in quieting the mental unrest, and in calling up such presentations as constitute the smallest opposition to the percept to be impressed.

This reminds us of the most important element among those which we still need in order to make of the preceding investigation a theory of attention—an element which is to be commended to the educator as one deserving his especial care. Even the faintest beginning of attention reproduces older presentations partly similar to, partly opposing, what has been noticed, and leading to opposite presentations. The ingredients and the strength of the reproduction are the results of earlier states of mind and of earlier culture. The educator, therefore, who requires attention without the proper preparation plays upon an instrument without strings.

To arrange the whole of instruction from its first beginnings to its conclusion most advantageously, so as to have each antecedent prepare the mental disposition of the pupil for each proximate and remote consequent, was

the problem I have made the main subject for consideration in several pedagogical writings. In respect to the preceding essay, what is said in the General Pedagogy concerning the alternation of immersion in a subject and reflection upon it as an ever-needful mental respiration may be restated as follows, though the restatement does not quite exhaust the meaning of those expressions: When a series of apperceptions has caused a certain summation of checks to accumulate, the latter should be allowed to subside before we attempt to go on. This law of the proper pausing in instruction, as we might call it, does not, however, contain the whole import of those words. To reflect is not merely to allow a summation of checks to subside. Reflection is fusion of what was first apperceived singly and by a separate consciousness. This is a subject for another psychological investigation, far more complicated than even the preceding. But this subject is not yet before me in an elaborated form. Neither is the closely connected subject of the reproduction of associated presentations, whose elaboration could alone enlighten us fully on such concepts as noticing and expecting, and therefore on the pedagogical art of continuously spinning on the thread of expectation, so as to set everything that has been noticed in the most correct relations to already existent expectations and to expectations still to be excited. The law of proper alternation alone receives from the preceding investigation a serviceable elucidation, thus: He who should tarry at what was wholly expected would meet with almost exhausted receptivity, because the presentation existent already in consciousness is capable of but little further gain. On the other hand, he who brings in the excessively novel, the totally foreign, has to fear the strong contrast which it will meet, and the strong summation of checks which it will form.

It may be left to thinking readers after understanding the preceding essay to give greater completeness as well as more definiteness to the reflections which are here indicated briefly. However, a pedagogue is guided not only by separate elaborated mathematico-psychological investigations, but even by the general metaphysical main view that such investigations are possible, so as to prevent his altogether missing the direction in the dark.

Such investigations can, of course, expect no applause from the adherents of the well-known teachers of transcendental freedom. All pedagogy costs them an inconsistency. The "intelligible" act of freedom in us does not stand in any temporal relation. On the other hand, education, when we leave out of thought its temporal beginning and progress and the causal relation between educator and pupil, becomes to us something completely incomprehensible. Pedagogy, therefore, is connected with a philosophy different from that of Kant, Fichte, or Schelling, and different even from that of Leibnitz; for his Pre-established Harmony would leave the educator and pupil no choice but to correspond with each other through the Deity.

The idea of a mathematical psychology, on the contrary, not only allows us to assume the possibility of acting upon the pupil, but also that to definite actions correspond definite results, and that by continued investigation and pertinent observation we shall more and more approach foreknowledge of the results. This has the especial advantage of removing an error to which practical educators are prone, usually to the same degree that they have a less accurate and familiar acquaintance with the idea of transcendental freedom. I refer to the notion that the talents which constitute what we call a man's turn of mind are an organic unit, unfolding in accordance

with inherent laws, to which, to be sure, nurture and food may be offered, but upon which can be imposed no other development than that which aboriginally is peculiar to it. This notion is favoured by experiences showing many a pupil to have grown into something wholly different from what parents and teachers intended. But such experiences only prove that the educators, completely losing the way in the darkness of psychological pedagogy, produced repugnance when their object was to produce inclinations and habits.

It is true, in the process of both enlarging and concatenating more intimately that which was already connected, every circle of thought and feeling becomes more and more similar to an organism which expels that which is repugnant to it, and assimilates the suitable elements it meets with. Nevertheless, there is no original organic constitution in the human soul any more than any other hypothesis of multiplicity will hold. All the more freedom remains to the educator's activity, who in early youth in large part himself forms the germ from which subsequently is produced what is apparently organic.

That put into general terms is the conviction which constitutes the basis of the idea of a mathematical psychology, and consequently of the hopes which may be thence transferred to pedagogy. A theoretical insight, not future but instantaneous, into the possibility of education, is, however, an impossible thought to one who sees before him mathematical psychology as a problem in the main unsolved.

Recently, none the less, a man has publicly affirmed that he possesses that theoretical insight into the possibility of education. Which is the philosophy of this man? Without inconsistency it can not be that of Leibnitz or Kant or Fichte or Schelling. But least of all is it mine;

for the same man in the same place has given a very extensive demonstration of what may become of a critique of my General Pedagogy with no vestige of knowledge of my Philosophy.*

The reader will the rather permit me to make a few observations in conformity with my philosophical convictions on the bright side of pedagogy, as they will enable me to conclude this article by pronouncing definitely to what extent I think pedagogy possible for the present.

In my General Practical Philosophy, second book, eighth chapter, I have indicated the scientific topic whence, out of the general and superior science, pedagogy, as far as subordinate to the latter, emerges. It will be understood that the content of the eighth chapter of the second book is determined by all that precedes, and that so extensive a disquisition could not be made an incidental appendix to a treatise on pedagogy. In the first place, the entire Theory of Ideas, or first book, is concentrated into the concept of virtue. In the next place, when the limitations and auxiliaries of man have been taken into consideration, this concept sets up side by side the problems in human culture and civic life. Of human culture, education is a pre-eminent part, and by accurately fitting itself to practical philosophy, pedagogy will in the latter find all the determinations of the pedagogic purpose completely together.

But even if pedagogy for the sake of popularity is not to fit itself closely to a presupposed systematic work, it must, nevertheless, know exactly the purpose toward which it is working. My General Pedagogy, though it appeared

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* We omit an extrémely long and rather personal foot note. It is the only omission which the translator has permitted himself.

earlier than the Practical Philosophy, was acquainted with the latter. The completed sketches of both, as well as the sketch of the Metaphysics, lay side by side. It was open to choice which was to be elaborated first. Precedence was given to that work which must necessarily, by reason of the lack of psychology, remain the less complete. The presentation was made as far as possible vivid and inciting to practice, and was so arranged as to let everybody meet first that which is more easily understood, and to put in, further on, texts at least for thoughts by the more patient readers. To remove, however, the possibility of anybody's fancying that the book pretended to be understood altogether by itself, the explanation of the main concepts was intentionally given with such aphoristic brevity as to make its insufficiency patent to everybody.

To Chancellor Niemeyer, eminently among others, we are indebted for excellent and detailed presentations of as much pedagogy as can be universally understood and is universally applicable. Clear moral concepts and an empirical psychology, not so much of systematic form as drawn from life, constitute the basis. This species of empiricism, enlarged by suitable experiments and combined with sharply determined concepts of Practical Philosophy, is doubtless the best pedagogy which, as a thoroughgoing work, homogeneous in all its parts, is possible hitherto. But let us hope that the time will come when it will be worth while to make the concept of virtue in the unity of its completeness the principal concept, and to inquire in the case of each of its requisites the means to the purpose from a speculative psychology which has stood the test of comparison with experience. Not until this happens—and the time for its happening is not as yet—shall we be able to boast that we possess a pedagogy which is in truth a science.

To our elementary exposition of Herbart's Pedagogy little need be added. We may say of Herbart's works what he said of Schwarz's Pedagogy: "When in an eminent writer full of heart and intellect we seem to miss something, it is competent for him to reply that if we will only let his work act upon us a longer time, if we will read ourselves into it, if we will use it anew and repeatedly on a variety of occasions, much will be found in it that is not set down in so many words. No work of significance can be more than one instance of intellectual riches that are far greater."

Of the intellectual wealth of Herbart not a tithe has been placed before the reader in this book. It has only been attempted to indicate the point of view which is right historically for surveying the Herbartian system and to impart an introductory knowledge, genuine and connected as far as it goes. But the reader should understand that he has only passed the outposts. Beyond lie the Outlines of Pedagogic Lectures and the General Pedagogy, whence he may make his way into the Text-book of Psychology and into the Psychology itself, not flagging by the way till he has entered the General Practical Philosophy also. If beyond these strongholds of Herbartianism he can penetrate the granitic Metaphysics of Herbart in all its stern realism, the main ridge of the Herbartian system, of which all the rest are but spurs, he will see something very different from what one familiar only with the idealistic philosophers of Germany means when he speaks of German metaphysics. The world will have changed for him, and he will know that education, the art of arts, is not a fortuitous assemblage of accidental devices, nor pedagogy, the queen of the sciences, to be wooed and won in a summer's day.

Not all the conclusions of Herbart may be accepted in all their details; yet the time has fully come when no one can pretend to an adequate knowledge of educational science unless he has, for a time, been the disciple of the greatest pedagogical thinker, who to the few that in his lifetime were able to appreciate his philosophic genius seemed "an unapproachable magnitude of the greatest content," and who

to those who knew his character was dear as one who "never knowingly hurt the feelings of a human being."

Nor is the latter qualification in Herbart inferior to the former. The Christian religion and the sociological philosophy of Herbert Spencer alike proclaim to us in the realm of thought and feeling what the spread of democracy and international arbitration show in the realm of facts: that the race "shall let the ape and tiger die"; that the time of altruism is coming; that education must produce not the clever man to whom morality is a convenient cloak, but the man whose cleverness is subservient to his ethical convictions. Thus men will go to Herbart, and having learned from the all-embracing pedagogist a pedagogy beginning with triangles and ending with the moral law, they will part with him bearing upon their minds the impress of the noblest line even that fine spirit ever penned—

"Love is the beautiful soul of life."

FIRST TABLE.—DETERMINATION OF

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